

A Complex Dynamics Approach to Black Holes

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Abstract

We propose a mathematical framework for modeling black hole horizons through complex dynamics, using Regge calculus to connect discrete curvature with stability multipliers in complex iterative maps.

Keywords: Black Holes, Mathematical, Bounded Orbits, Complex Dynamics

1. Introduction

Black holes represent some of the most fascinating and extreme objects in our universe. Recent theoretical developments suggest that the near-horizon region of black holes may possess a *fractal-like structure*, reflecting complex self-similar patterns in spacetime. Fractal models provide a rich language to explore how black hole geometries evolve and affect surrounding fields. In this context, the tools of *complex dynamics* and fractal iteration theory offer promising avenues to model black hole escape regions and horizons. Through the lens of complex iterative functions, one can construct analogies that map the Schwarzschild radius to fractal set boundaries characterized by stability multipliers λ . This connects the mathematical theory of Mandelbrot and Julia sets to physical horizons, suggesting a deep geometric and dynamical correspondence [1,2,4-9].

2. Main

2.1. Continuous Complex Iterative Map

We begin by constructing a modified iterative map inspired by the Schwarzschild geometry, such continuous model would have used the Schwarzschild metric that contains the factor: $g_{tt} = 1 - \frac{2GM}{rc^2} = 1 - \frac{r_s}{r}$ where $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius. This factor vanishes at $r = r_s$, marking the event horizon. To incorporate this into a complex dynamical system, one can easily define a modified iteration inspired by the Mandelbrot set. For $z = x + iy \in \mathbb{C}$, the radial modulation function will be: $\psi(r) = \sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{1 - \frac{r_s}{r}}$ has singularity at $r = 0$. Since such work is in the complex plane one can define distance like radii as $|z| = \sqrt{x^2 + y^2}$ and since $|z|$ is the distance scaling to the horizon one can plug it instead of r get $\psi(|z|) = \sqrt{1 - \frac{r_s}{|z|}}$. Then one would have plug it on the Mandelbrot set as the factor that replaces z to get $f(z) = z \cdot \psi(|z|) + c$. One can see in Fig. 1 a circular "attracting" shape that 1 corresponds with observational physics. The main problem in such method is that this continuous map is *not holomorphic*, limiting the applicability of standard complex dynamics theorems.

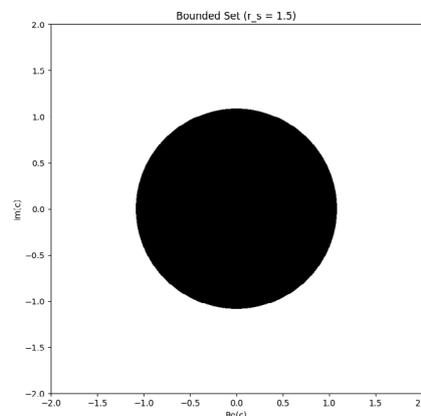


Figure 1: Dichotomous Visualization of the Schwarzschild Fractal Map. Black Points Represent Bounded Orbits, White Points Represent Escape. The Circular Boundary Corresponds to the Schwarzschild Horizon

3. Discrete Complex Iterative Map

To overcome the issue of non-holomorphic maps we will suggest our main aim in this paper, a Regge Calculus approach to complex iterative functions. Regge calculus discretizes spacetime where curvature is concentrated on deficit angles at n -dimensional faces, this creates a natural correspondence with complex dynamics deficit angles, where ϵ act as discrete curvature measures that can be directly related to the stability multipliers λ of complex dynamics maps. We will use Regge's original ϵ -cone which represents the fundamental building block of discrete spacetime geometry. The ϵ -cone concentrates all curvature at one vertex, hence a single vertex singularity. The metric, as defined by Regge, of a 4-dimensional system is [10].

$$ds^2 = dz_1^2 + dz_2^2 + d\phi^2 + \phi^2 d\theta^2 \quad (1)$$

provides the foundational geometry. Based on Regge's axioms 1 – 4 an Euclidean based structure that is differentiable maintains smoothness away from the vertex, enabling holomorphic analysis [10]. The key insight is that this single vertex encapsulates the entire gravitational field geometry of the black hole with deficit angle ϵ concentrated at the central vertex. The Regge action, expressed as a sum of deficit angles weighted by hinge areas [10].

$$S = \sum_h A_h \epsilon_h \quad (2)$$

encodes the discrete curvature distribution of the simplicial spacetime. By focusing on the variation of this action with respect to edge lengths $\sum_h \frac{\partial A_h}{\partial l_i} \epsilon_h = 0$ and translating the resulting geometric constraints into the complex plane, the parallel transport around the hinge corresponds to a holonomy rotation by ϵ hence a multiplication by $\exp(i\epsilon)$ in our means, yielding the map.

$$f(z) = \exp(i\epsilon) \cdot z + c \cdot g(z) \quad (3)$$

Where ϵ is the deficit angle, $g(z)$ is a holomorphic function encoding the cone geometry and c is the standard complex parameter in complex iterative maps. As for the cone geometry $g(z)$, we will plug the fundamental solution to the Laplace equation $\nabla^2 u = 0$ in a 4-dimensional system $g(z) = \frac{1}{(z-z_0)^2}$, which naturally encodes the local geometric influence of the conical singularity, hence the full iterative function is.

$$f(z) = \exp(i\epsilon) \cdot z + \frac{c}{(z-z_0)^2} \quad (4)$$

where ϵ encodes the deficit angle or curvature strength around the singularity $(z-z_0)$. The linearization around any point $z=z_0$ gives $f'(z) = \exp(i\epsilon) - \frac{2c}{(z-z_0)^3}$ and for points far from the singularity $f'(z) \approx \exp(i\epsilon)$ so $|\lambda| = 1$ and the argument is ϵ which perfectly matches the holonomy interpretation in Regge's ϵ -cone where Levi-Civita parallel transport around the vertex produces rotation by ϵ , a multiplication by $\exp(i\epsilon)$ [3]. A natural connection arise between the deficit angle and the stabilizer

$$|\lambda| = \exp(i\epsilon) \quad (5)$$

In this discrete geometric framework, spacetime is modeled as a piecewise flat Lorentzian manifold constructed from simplicial building blocks. Each simplex carries a locally flat metric of signature $(-, +, +, +)$ consistent with the Lorentzian structure of general relativity. Curvature is not distributed uniformly but is concentrated on "bones" where the sum of dihedral angles deviates from the flat space value. These deficits ϵ represent intrinsic curvature and generalize the concept of angular deficit from 2-dimensional cones to 4-dimensional spacetimes. The bones themselves are classified as timelike, spacelike, or null according to the character of their tangent vectors, with corresponding real or imaginary deficit angles. This Lorentzian simplicial complex provides a natural arena for a discrete variational principle: the Regge action which is defined as the sum over all bones of the Lagrangian $S = \sum_n \mathcal{L}_n = \sum_{bones\ n} \epsilon_n L_n$ where L_n is the Lagrangian which is given by the product of the deficit angle ϵ and the Lorentzian invariant L_n of the bone n . Varying the edge lengths l_i yields discrete Einstein-like equations governing the geometry [10]. Integrating this Lorentzian structure into the suggested complex dynamical framework allows the encoding of discrete holonomies as rotations by ϵ realized via complex phases $\exp(i\epsilon)$ in a manner that respects causal and metric structure, thereby bridging Regge's geometric insight with the analytic power of holomorphic iteration.

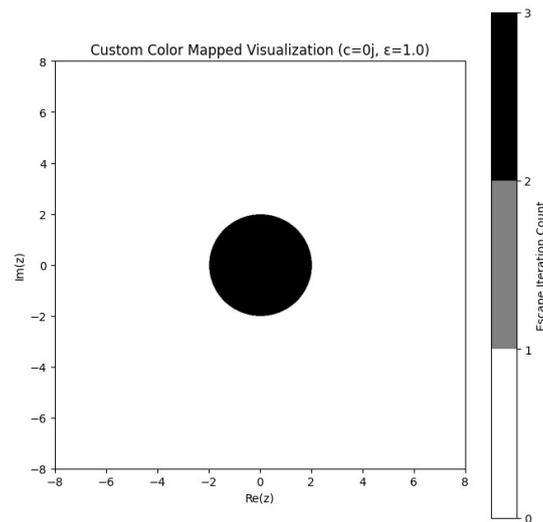


Figure 2: Dichotomous visualization of the Schwarzschild fractal map. Black points represent bounded orbits, white points represent escape. The circular boundary corresponds to the Schwarzschild horizon with any deficit angle ϵ . For various complex c one gets almost circular boundaries.

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