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Mathematical Modeling of Mechanical Thermodiffusion Processes Under Influence of Microwave Irradiation

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Abstract
In this work the methods of applying the spatial averaging method to the description of the diffusion, electrodynamics and mechanical properties of a porous multiphase (composite) wetted medium are considered. The set of physical phenomena and processes in porous wetted materials is described in detail, as well as mathematical or theoretical models are proposed to describe the interaction of moisture, electromagnetic and mechanical fields. For a porous humidified medium, a closed system of equations was obtained, which connects the processes of heat and mass transfer with internal heat sources induced by external microwave electromagnetic radiation and mechanical stresses within the framework of the model of a continuous medium.

Keywords: Microwave Irradiation, Porous Media, Heat and Mass Transfer, Mechanical Stress

1. Introduction
Microwave technologies are widely used in industrial processes, such as drying (textiles, wood, paper and ceramics), heat treatment of metals (hardening), disposal (recycling or disposal) of radioactive waste. In medicine, they are used to restore frozen tissues, warm blood, and treat tumors. The greatest consumer interest in microwave technologies arises in the food preparation industry, namely the processes of baking, pasteurization, dehydration and sterilization. Volumetric heating of the material by microwave irradiation (internal dielectric heating) refers to accelerated methods of heat exchange and removal of moisture from the material. For various methods of dielectric heating, the prediction (forecasting) of heat and mass transfer (transport) processes is extremely important in terms of equipment development, optimization processes, and product quality improvement. Direct experimental measurement of temperature and moisture content in porous material is a complex and troublesome process. Therefore, a significant amount of scientific research was carried out to model the transport (diffusion) processes of heat and mass transfer for such heterogeneous materials. Several different techniques or methods of numerical calculations, which are based on the use of the finite difference method finite elements or the line transfer matrix method have been used to simulate microwave heating with varying degrees of success [1-4]. In fact, when performing these calculations, it is necessary to know the constant dielectric or thermal properties of the material [5-10].

A lot of research has been done in the modeling of microwave heating of materials, but we have little progress in modeling the transfer of heat and mass during microwave drying [11]. Since under influence of dielectric heating or drying in an electromagnetic field, the material with dielectric loss of power of microwave irradiation has no permanent electro-physical properties, for most materials the dielectric properties change as a function of moisture content and temperature. Therefore, during microwave heating (drying), the distribution of the electromagnetic field in the body (material) is strongly related to the processes of heat and mass transfer. Changes in the local moisture content and temperature affect to the dielectric properties of the material, including the distribution of the electromagnetic field. The size of the load (material) relative to the waveguide or the drying cavity, the effect of the wave resistance (impedance) of the cavity, the amount of the radiation power that is reflected from the inner walls of the cavity (resonator) in the direction of the on magnetron determine the quality of the microwave equipment.

Since the detailed distribution of the power of the electromagnetic field (internal heat sources) within the material loading is quite difficult to model, most studies assume that the lines of force of the electromagnetic field during microwave irradiation on the surface of the material are uniform and normal to the surface [4]. It is also assumed that the power of the external irradiation in the body decays in accordance with the exponential law [12]. Such approach was happily applied to modelling of heat and mass transport of dielectric the heated product of food [13,14]. A simple reflection was expressed according to investigation, where heat sources under dielectric losses of microwave irradiation in the range of solid product is assumed uniform [15]. In reality, into the mentioned theoretical works the understanding of drying processes is limited, because under description of humidity transfer in porous media at isothermal conditions is used in general. For more developed models a law of molecular diffusion is applied and assumed, that heat gradient is the main division force into processes of mass transport or transfer [16-18]. It's need to pointed, that results of numerical modelling according to mentioned above models for humidity diffusion as rule do not agree with experimental measured in the more cases [19].

Same researcher development of the models into approach, when diffusion of water vapour is main mechanism of humidity transport during of drying processes [20]. For too refined methods for describing of drying (microwave heating) the two area transport models are treated [21-23]. In these models is assumed, that two different area (region) for describing of humidity transport during drying (wetted and dried area) is existed. In the wetted area, moisture content is great as maximal sorption value for water vapour and basic mechanism for transport (transfer) of humidity is movement of liquid. In sorption region, the movement of adsorbed (bounded) water and water vapour is the basic reason of mass transport. The applying of this model is limited, because during the long (limited) process of drying the division (boundary) between wetted and sorption areas (regions) is conventional. A reference review is shown that main progress was made into development of transport (diffusion) models under investigation of physics of ground, where main interest was concentrated on the utilization of nuclear remains and management of water resources [24]. The basic formulation was development for interconnected processes of heat and mass transfer into works [25-27]. The main approaches are based on assumption, that general transport potential consist on the two components: temperature and capillary potential.

During microwave drying an induces by thermal (microwave) heating increment in density of water vapour and humidity potential is calling of movement for humidity from more heated into cooled area. A general restriction or deficiency of mentioned isothermal models is absence of the thermal induced changing of humidity. First, this is due to a significant difference in the distribution of moisture in the porous material, which is caused by the processes of evaporation or condensation.

In fact, in the context of drying processes, reformulation of the problem in terms of moisture content and temperature is desirable for a fundamental understanding of drying processes, especially when modelling diffusion (transport) processes under conditions of intense internal microwave heating. Foremost, this is due to a significant difference in the distribution of moisture in the porous material, which is caused by the processes of evaporation or condensation.

The nature of stresses in porous materials that arise during microwave heating, as a result of changes in temperature and moisture distribution, also remains insufficiently researched. For example, despite the fact that the hydrothermal behavior of concrete under thermal (radiation) heating is described in detail in the work, the author is aware of only one scientific work, where the maximum values of thermal stresses caused by internal microwave heat sources were calculated in the specified material [28,29]. Some attempts at computer simulation of the stressed state of the specified material under microwave irradiation were carried out in works, where only the energy (thermal) balance equation was used to calculate the thermal and elastic properties of the material [30,31]. Special attention should be paid to the study of the properties of thermal and moisture stresses during microwave irradiation under conditions of phase transformation (evaporation or condensation) and changes in the pressure of the gas medium into the pores of the moistened material. Theoretical models for describing the dependence of microwave heating sources on the distribution of moisture and temperature in a porous medium also need improvement and development, especially in view of modelling effective (measured) characteristics due to the corresponding properties of phases or components in the studied wetted porous sample.

1.1. Model and analysis

Interaction of the material with the electromagnetic field for a non-magnetic dielectric medium with conductive properties adsorbs the energy of the electromagnetic field and converts it into the heat.

In the microwave or dielectric ranges of frequency (Fig.1) area the properties of the material with dielectric losses are determined by the ratio

$$\varepsilon = \varepsilon' - \varepsilon'', \qquad (1)$$

here ε is the complex dielectric constant, ε' is the relative dielectric constant, ε'' is the dielectric loss factor, also

$$tg\delta = \varepsilon'' / \varepsilon' , \quad (2)$$

is the tangent of the dielectric loss angle.

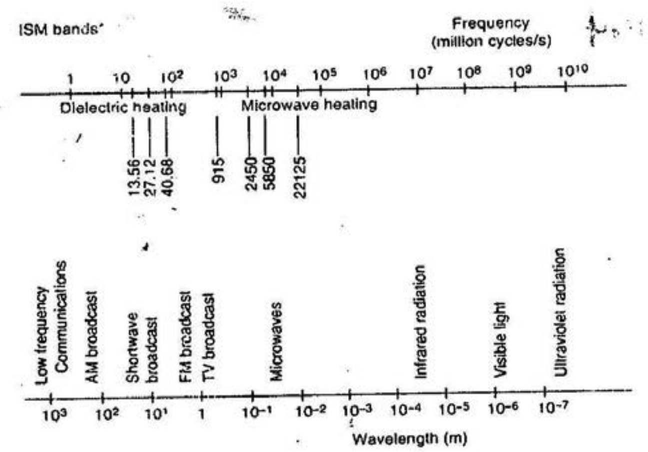

Figure 1: The Electromagnetic Spectrum

The main dielectric losses are linked with properties of free (not adsorbed or non-bounded) water, as shown on Fig.2 below

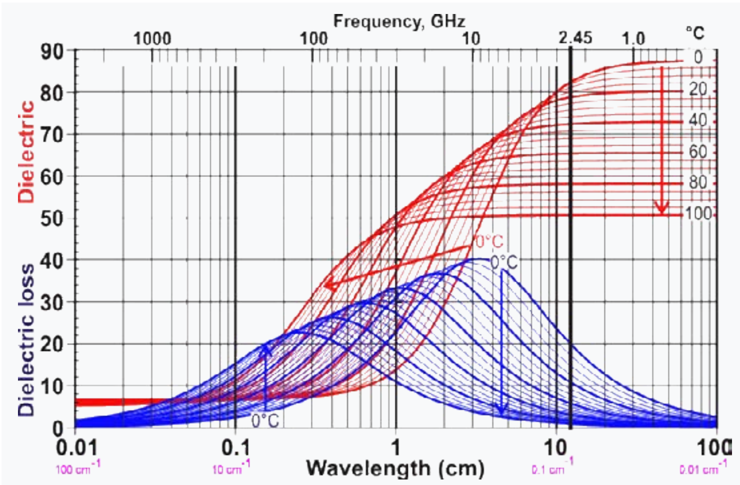

Figure 2: The Dielectric Properties of Free Water in the Specific Spectral Interval

Where are any other types of dielectric relaxation (see Fig.2) may to meets in the porous body. We are considering only dipole rotation under condition that scattering according to electric conductivity or joule heat release are negligees.

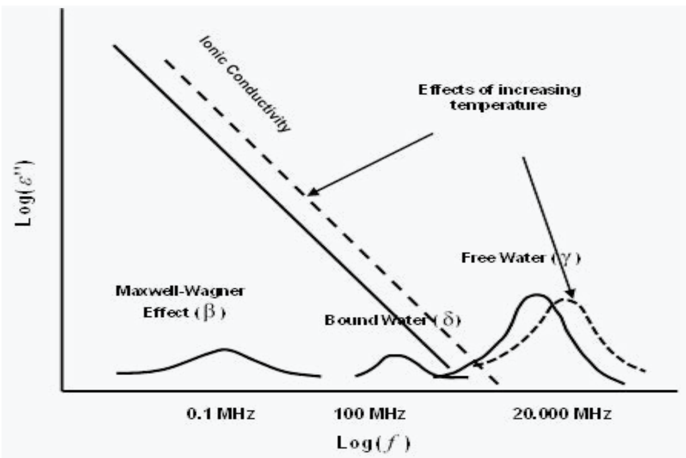

Figure 3: Types of Dielectric Relaxation in the Porous Wetted Material

Comparing to convective drying, we can point to advantage of microwave or dielectric drying: 1. High energy efficiency in the period of falling speed, which is a consequence of the concentration of the energy of heat release in places of liquid concentration; 2. A significant increase in fluid mobility with an increase in the internal pressure of water vapour due to the effect of liquid evaporation; 3. Significant improvement or preservation of the quality of the drying product due to the near-uniform distribution of heat sources, which corresponds to the insignificant distribution of stresses along the thickness of the body; 4. Significant improvement or preservation of the quality of the drying product due to the near-uniform distribution of heat sources, which corresponds to the insignificant distribution of stresses along the thickness of the body; 5. The duration of convective drying can be significantly reduced in accordance with the expediency of saving energy costs.

The acceleration of convective drying applying microwave is demonstrated on the Fig.4 below. There is the significant time saving in drying according to time point of applying microwave. In other word, it's possible to combine convective and microwave drying for reduction of drying period.

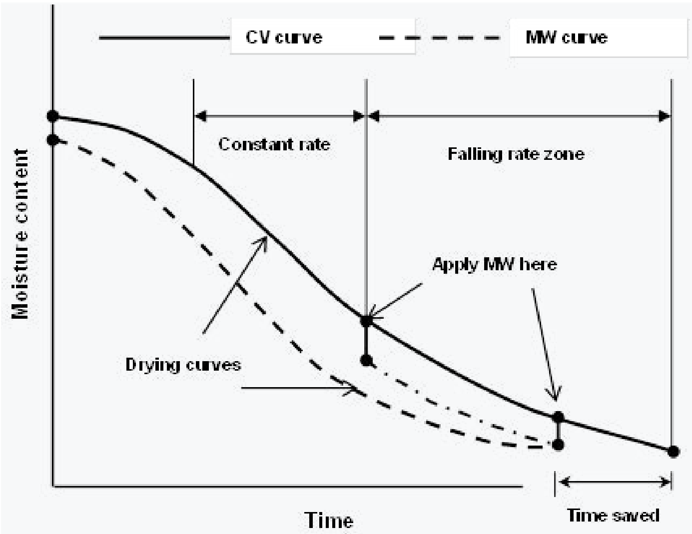

Figure 4: Curves of Mixed Convective and Microwave Drying

There are also differences into nature of destruction for investigate porous material, as it is depicted on the Fig.5 below

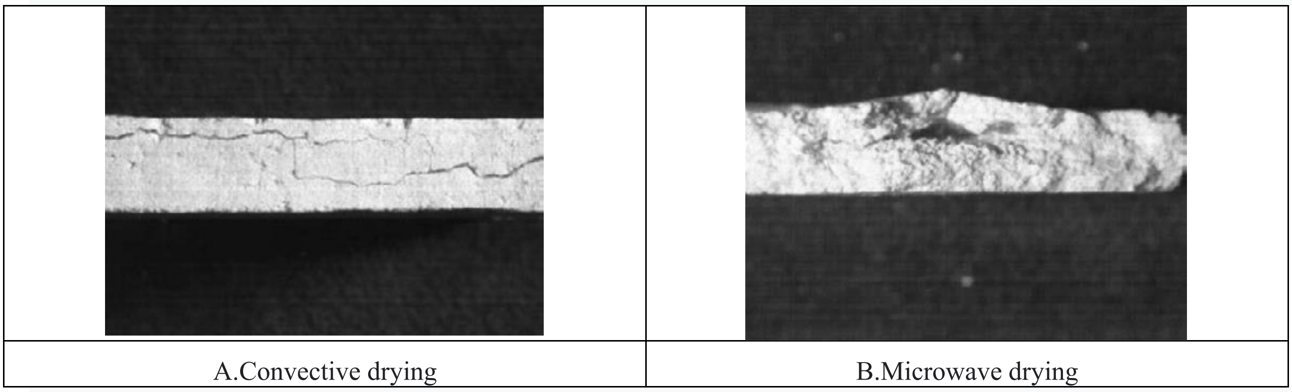

Figure 5: Examples of Destruction of Ceramics Under Convective (A) and Microwave (B) Drying

The main goal of this work to divide of mechanical, electromagnetic and diffusional processes to correct describes of the mentioned physical properties of wetted (humidified) porous material under influence of the symmetric microwave irradiation for the one-dimensional case into approach of continues media. In general, allow me to characterize and name these phenomena as mechanical Thermodiffusion in the proposed terminology.

1.2. The object of studies

Lets define the object of studies. This is one dimensional porous wetted plate length of L (Fig.6) which is under influence of symmetrical microwave irradiation of the fixed power. The constant convective flow by the hot air at the surfaces of plate is also additionally applied during all time of microwave treatment, so in general it is can be classified as the mixed microwave-convective drying.

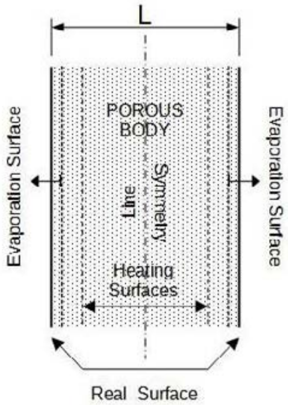

Figure 6: The Modelling of Microwave Treatment of Humidified Porous Sample Under the Symmetric Boundary Conditions

We are modelling such drying system mathematically via introduction of dynamical boundary conditions. There is confirmed by other authors that, conditionally, it is possible to highlight the two zones during of microwave treatment for porous sample: wetted and heated zone [21-23]. The term zone is meaning as a one-dimensional area from the internal surface to the real hard boundaries of body into the direction of external environment. The moving that zones is interconnected due to dependence of diffusion coefficients in the internal volume of sample from humidity, as well as the time dependences of surface heat and mass transfer coefficients on the real physical boundaries of mentioned porous sample.

To describe the diffusion processes in a porous plate, the well-known averaged heat and mass transport equations written in general form are used

$$\left(\rho C\ \right)_{eff}\partial\ T = \partial_x\left[\lambda\quad \partial\ T\right] - \langle r\rangle\,\dot{I} + \dot{q} \qquad (3)$$

$$\partial_t\langle c_L\rangle \qquad \langle\vec{J}_L\rangle \qquad \dot{I} \qquad\qquad \partial_t\langle c_v\rangle + \qquad \langle\vec{J}_v\rangle \qquad \dot{I}$$

$$\partial_t\langle c_a\rangle + \partial_x\langle\vec{J}_a\rangle = 0$$

where $\left(\rho C_p\right)_{eff} \quad \sum_\sigma \theta\ \ \langle\rho_\sigma\rangle^\sigma C_p^\sigma$ and $\lambda\ \ _f = \sum_\sigma \theta_\sigma\,\langle\lambda_\sigma\rangle^\sigma$,

here $\sigma = \{S,L,G\}$ are the effective thermal characteristics of porous material, $\dot{q}$ and $\dot{I}$ are the heat and mass surfaces intensity correspondingly, C_p^σ and $\langle\lambda_\sigma\rangle^\sigma$ are the specific capacity and thermal conduction of σ -phase accordingly and $\langle \mathbf{r}\rangle$ is the latent heat of vaporization [32,33].

There are the expressions for concentrations

$$\langle c_L\rangle^L = \langle\varphi\rangle\eta_L\langle\rho_L\rangle^L,\ \ \langle c_v\rangle^G = \langle\varphi\rangle(1-\eta_L)\langle\rho_v\rangle^G,\ \langle c_a\rangle^G = \langle\varphi\rangle(1-\eta_L)\langle\rho_a\rangle$$

and flows of phases (components) in the porous media

$$\langle J_L^x\rangle = -\langle\rho_L\rangle^L\frac{k_i}{\mu}\partial_x P_L,\ \ \langle J_\alpha^x\rangle = -\langle\rho_\alpha\rangle^G\frac{{}_{in}k}{\mu}\partial_x P_G - \langle\rho_G\rangle^G D_{eff}\partial\left[\langle\rho_v\rangle^G/\rho_G\right]$$

here $\psi_L = [k_{in}\ \ k_{rL}\ ] / \mu_L$ and $\psi_G = [k_{in}\ \ k_{rG}\ ]/\mu_G$, where k_{in} and $k_{r\alpha}$, $\alpha = \{L, G\}$ the intrinsic permeability of skeleton and liquid phase correspondingly, $\langle\rho_L\rangle^L = \rho_L$ $\langle\rho_\alpha\rangle^G = p_\alpha\ \ M_\alpha /$ RT where $\beta = \{v, a\}$ is the corresponding densities of liquid phase and components of gas mixture, μ_L and μ_G is dynamical viscosity, $D_{eff} = \left[(M_v M_a)/M_G^2\right]D$ is model diffusion coefficient, P_L and P_G is the values of pressures in liquid and gas phases, P_C is a capillary pressure, M_v and M_a is the molar mass of water vapour and dry air respectively.

Constitutive or material equations: 1. Pressure balance into liquid and gases phase: $P_L = P_G - P_C$, where $P_C(\eta_L) = \sigma\sqrt{(k_{in}/\langle\varphi\rangle)}$ J(η_L) is capillary pressure, $J(\eta_L)$ is dimensionless characteristic of porous material named as J-Laverett [34] function; 2. The

Daltone's law: $p_v + p_a = P_G$, where p_β is the partial pressures of gas mixture components ($P_G = \rho_G$ RT / M_G here M_G = 1/ ([$\langle\rho_v\rangle^G / \langle\rho_G\rangle^G$]/M_v +[$\langle\rho_a\rangle^G/\langle\rho_G\rangle^G$]/M_a)) a is the molar mass of mixture); 3. Equation of sorption equilibrium $P_c = -\rho_L$ RTLn [ϕ], where $\phi = p_v / p_{vs}$ is the relative humidity and p_{vs} is the partial pressure of water vapour into the gas phase.

Modification of heat and mass equations: 1. A transition from variables has been made from {η_L, T, P_G} to {η_L, T, x_v} according to Rault's law [32] $x_\alpha = p_\alpha / P_G$ ($x_v + x_a$ = 1), where x_α is molar fraction of components, the intensity of evaporation sources in the internal volume and at the surface of the plate is determined in the different ways

$$\dot{I}^{(int)} = -\frac{1}{2}(\partial_t[\langle c_L\rangle^L - \langle c_v\rangle^G] + \partial_x[\langle J_L^x\rangle + \langle J_v^x\rangle])$$

$$\dot{I}^{(ext)} = -\frac{(\rho C_p)_{\text{eff}}}{\langle r\rangle}\partial_t T + \frac{1}{\langle r\rangle}\partial_x\left[\langle\lambda_{eff}\rangle\partial_x T\right] + \frac{\dot{q}}{\langle r\rangle}$$

2. An expression for the effective Burger diffusion coefficient of the gas mixture in the pores of the material is proposed

$$D_G^{\text{eff}} = \frac{1}{\tau}\frac{\langle\varphi\rangle(1-\eta_L)}{x_v - \langle\varphi\rangle(1-\eta_L)(x_v - 1)}D_v^a, \quad (5)$$

where D_v^a is the diffusion coefficient of water vapour into dry air, τ a is the tortuosity factor.
3. For the internal volume

$$(\rho C_p)_{\text{eff}} - \frac{1}{2}\langle r\rangle\partial_t[\langle c_L\rangle^L - \langle c_v\rangle^G] = \partial_x\left[\langle\lambda_{eff}\rangle\partial_x T\right] + \frac{1}{2}\partial_x[\langle J_L^x\rangle + \langle J_v^x\rangle] + \dot{q}$$

$$\partial_t[\langle c_L\rangle^L + \langle c_v\rangle^L] = -\partial_x[\langle J_L^x\rangle + \langle J_v^x\rangle], \quad \partial_t\langle c_a\rangle^G = -\partial_x\langle J_a^x\rangle \quad (6)$$

and surface of plate

$$\partial_t\langle c_v\rangle^L + \frac{(\rho C_p)_{\text{eff}}}{\langle r\rangle}\partial_t T = -\partial_x\langle J_L^x\rangle + \frac{1}{\langle r\rangle}\partial_x\left[\langle\lambda_{eff}\rangle\partial_x T\right] + \frac{\dot{q}}{\langle r\rangle}$$

$$\partial_t\langle c_v\rangle^G - \frac{(\rho C_p)_{\text{eff}}}{\langle r\rangle}\partial_t T = -\partial_x\langle J_v^x\rangle - \frac{1}{\langle r\rangle}\partial_x\left[\langle\lambda_{eff}\rangle\partial_x T\right] - \frac{\dot{q}}{\langle r\rangle} \quad (7)$$

$$\partial_t\langle c_a\rangle^G = -\partial_x\langle J_a^x\rangle$$

the systems of heat and mass transport equations are separated [35].

Modelling of microwave treatment processes: Let's define the real body surfaces as S (see Fig.1), heating and evaporation surfaces as S_1 and S_2 respectively.at the initial moment of time. The near-surface zones of wetting and heating of the material are considered, delimited by the surfaces S_1 and S_2, within which the solutions of the system of equations and the power of the microwave heating sources are assumed to be constant.

Internal diffusion processes: The modified system of equations (6) and (7) with a defined effective diffusion coefficient (5) of the gas mixture in the pores of the moistened material with corresponding expanded expressions for the flows is considered

$$\langle J_L^x\rangle = \psi_L\rho_L\left(\frac{\partial p_c}{\partial\eta_L} - \frac{1}{x_v}\frac{\partial p_v}{\partial\eta_L}\right)\partial_x\eta_L - \psi_L\rho_L\left(\frac{\partial p_c}{\partial T} - \frac{1}{x_v}\frac{\partial p_v}{\partial T}\right)\partial_x T + \psi_L\rho_L\frac{P_G}{x_v}\partial_x x_v$$

$$\langle J_v^x\rangle = -\psi_G\frac{1}{x_v}\langle\rho_v\rangle^G\frac{\partial p_v}{\partial\eta_L}\partial_x\eta_L - \psi_G\frac{1}{x_v}\langle\rho_v\rangle^G\frac{\partial p_v}{\partial T}\partial_x T + \left(\psi_G\frac{P_G}{x_v}\langle\rho_v\rangle^G - D_{\text{eff}}\langle\rho_G\rangle^G\right)\partial_x x_v \quad (8)$$

$$\langle J_a^x\rangle = -\psi_G\frac{1}{x_v}\langle\rho_a\rangle^G\frac{\partial p_v}{\partial\eta_L}\partial_x\eta_L - \psi_G\frac{1}{x_v}\langle\rho_a\rangle^G\frac{\partial p_v}{\partial T}\partial_x T + \left(\psi_G\frac{P_G}{x_v}\langle\rho_a\rangle^G + D_{\text{eff}}\langle\rho_G\rangle^G\right)\partial_x x_v$$

The thicknesses of the surface zones are determined by the ratios

$$\Delta h_1 = \frac{1}{h_m}\psi_G\frac{p_v}{x_v}_{|S_0}, \qquad \Delta h_2 = \frac{1}{h_T}\lambda_{\text{eff}|S_0}$$

where h_m and h_T are the constant surface mass and heat exchange coefficients under convective heating conditions, S_0 is a surface of symmetry of the porous plate.

Flow balances: Is defined by equalities for liquid $\langle J_L^x\rangle_{|S_1^-} = J^*_{L|S_1^+}$ and components $\langle J_v^x\rangle_{|S_1^-} = J^*_{v|S_1^+}$ and $\langle J_a^x\rangle_{|S_1^-} = J^*_{a|S_1^+}$ of gas mixture .

Boundary conditions: Are defined relative to transport flows of mass on the surface S_0 under additional conditions: 1. There is no liquid flow ($P_{L|S_1} = 0$) $P_{C|S_1} = P_{G|S_1}$ and the transformation $\langle\rho_L\rangle^L{}_{|S_1} = \langle\rho_v\rangle^G{}_{|S_1}$ of the liquid into water vapour on the boundary surfaces S_1, $x = \{(\text{-L}+\Delta h_1)/2, (\text{L-}\Delta h_1)/2\}$ of constant wetting occurs instantly; 2. Heat sources in the near surface areas $x \in [-\text{L}/2, -\text{L}/2 + \Delta h_2] \cup [\text{L}/2\text{-}\Delta h_2, \text{L}/2]$ of plate heating are uniform.

Than

$$J_L^* = -\frac{h_T^*}{\langle r\rangle}\left(T_{|S_1} - T_{|\text{amb}}\right),$$

$$J_v^* = -\langle\varphi\rangle(1-\eta_L)\frac{\langle\rho_a\rangle^G}{\langle\rho_v\rangle^G}h_m^*\left(\langle\rho_v\rangle^G{}_{|S_1} - \langle\rho_v\rangle^G{}_{|\text{amb}}\right) - \frac{1}{T}\langle\rho_a\rangle^G\frac{D_v^a}{\lambda_G}\frac{h_T^*}{\langle r\rangle}\left(T_{|S_1} - T_{|\text{amb}}\right) +$$

$$+\langle\varphi\rangle(1-\eta_L)\frac{1}{x_v}\langle\rho_a\rangle^G\left(h_m^* + \frac{x_v}{1-x_v}\frac{M_v}{M_g}D_S^a\right)\left(x_{v|S_1} - x_{v|\text{amb}}\right),$$

$$J_a^* = -\langle\varphi\rangle(1-\eta_{sL})h_m^*\left(\langle\rho_v\rangle^G{}_{|S_1} - \langle\rho_v\rangle^G{}_{|\text{amb}}\right) -$$

$$-\left(\langle\rho_v\rangle^G\frac{D_v^a}{\lambda_G}\frac{1}{T} + 1\right)\frac{h_T^*}{\langle r\rangle}\left(T_{|S_1} - T_{|\text{amb}}\right) + \langle\varphi\rangle(1-\eta_L)\frac{1}{x_v}\langle\rho_v\rangle^G\left(h_m^* - \frac{M_a}{M_g}D_S^a\right)\left(x_{v|S_1} - x_{v|\text{amb}}\right),$$

here $h_m^* = \left[\psi_G\frac{p_v}{x_{v|S_1}}/\psi_G\frac{p_v}{x_{v|S_0}}\right]h_m$ and $h_T^* = \left[\lambda_{\text{eff}|S_1}/\lambda_{\text{eff}|S_0}\right]h_T$ are the variable with time heat and mass transfer coefficients, $D_S^a = \langle\varphi\rangle^{1/3}(1-\eta_L)^{7/3}D_v^a$ is the known Millington-Quirk [36] coefficient of diffusion for dry air component in the pores of material at surface S_2 of evaporation.

1.3. Electromagnetic interaction

As was described in the paper for a flat plate into the electric field strengths, the wave equation will have the form

$$\partial_x^2\langle\vec{E}_*^t\rangle(x) + k_0^2\left[\bar{n}_\omega^{eff}(x,t)\right]^2\langle\vec{E}_*^t\rangle(x) = 0,$$

here $\bar{n}'_\omega(x,t) = \frac{\bar{k}'_\omega(x,t)}{k} = \sqrt{\bar{\varepsilon}_\omega^{eff}(x,t)}$ is the complex refractive index, $\bar{k}'_\omega(\vec{x},t)$ is the wave vector into the porous (unhomogeneous) media, $k_0 = \omega\sqrt{\mu_0\varepsilon_0} = \omega/c_0$ (where $c_0 = 1/\sqrt{\mu_0\varepsilon_0}$ is the light velocity in vacuum) is the wave vector of electromagnetic irradiation in the vacuum, $\omega = 2\pi f$ is the corner frequency electromagnetic field (f is the lineal frequency), μ_0 and ε_0 are the magnetic and electric constants in the vacuum correspondingly [37].

We are found the solution of the wave equation as it is specified into the work according to the method of W.K.B. (Wentzel-Kramers-Brillouin) in the form

$$\langle E_{*y}^t\rangle(x) = \frac{1}{\sqrt{\bar{n}_\omega^{\text{eff}}(x,t)}}\left(\begin{array}{c} A(t)\sqrt{\bar{n}_\omega^L(t)}\text{Exp}\left[-\text{ik}_0\int_{-L/2}^{x}\bar{n}_\omega^{\text{eff}}(x,t)\,\text{dx}\right] + \\ +B(t)\sqrt{\bar{n}_\omega^R(t)}\text{Exp}\left[\text{ik}_0\int_{L/2}^{x}\bar{n}_\omega^{\text{eff}}(x,t)\,\text{dx}\right]\end{array}\right)$$

where $\bar{n}_\omega^\alpha(t) = \bar{n}'_\omega(\pm L/2, t), \alpha = \{L, R\}$ are the constant values of the refractive index on the edges of porous plate, $A(t)$ and $B(t)$ are the unknown functions of time [37,38].

Under applying of boundary conditions to the components and derivatives of electro-magnetic field at the edges of the plate we get the time-varying coefficients

$$A(t) = E_L \frac{T_L^t - (E_R/E_L)\delta(t) T_R^t R_L^t \gamma_{RL}(t)}{1-[\delta(t)]^2 R_L^t R_R^t},$$

$$B(t) = E_R \frac{T_R^t - (E_L/E_R)\delta(t) T_L^t R_R^t \gamma_{LR}(t)}{1-[\delta(t)]^2 R_L^t R_R^t},$$

where $T_\alpha^t = 2/[1+\bar{n}_\omega^\alpha(t)]$ and $R_\alpha^t = [1-\bar{n}_\omega^\alpha(t)]\ /[1+\bar{n}_\omega^\alpha(t)]$ (here $\alpha = \{L, R\}$ is the index, which corresponds to L and R edged of plate), are respectively the coefficients of transmission and reflection of a plane wave at the boundaries of the plate in the accepted notations [37].

In the approximation of large plate L thicknesses according to for the power of microwave irradiation into the porous plate we have

$$\langle P_x^t \rangle(x) = P_{\text{in}} |\bar{n}_\omega(t)| |T(t)|^2 \text{Re}\left[\sqrt{\frac{\tilde{\bar{n}}_\omega^{\text{eff}}(x,t)}{\bar{n}_\omega^{\text{eff}}(x,t)}}\right] \times$$
$$\times \left(\text{Exp}\left[-2\int_{-L/2}^{x} \alpha(x,t)\,\text{dx}\right] + \text{Exp}\left[-2\int_{x}^{L/2} \alpha(x,t)\,\text{dx}\right]\right)$$

where $P_{\text{in}} = E_0^{\ 2}/2\,\eta_0$ is the power of external microwave irradiation, $\bar{n}_\omega(t) = \bar{n}_\omega^{\text{eff}}(\pm L/2\,, t)$ and $T(t) = 2/[1+\bar{n}_\omega(t)]$ are the boundary values of refractive index and transmission coefficient for electromagnetic wave correspondingly [37].

The poser of heat sources of electromagnetic (microwave) heating is obtained by the relation

$$\dot{q}^t(x) = -\partial \langle P_x^t \rangle$$

and is getting the following view

$$\dot{q}^t(x) = 2P_{\text{in}} |\bar{n}_\omega(t)| |T(t)|^2 \alpha(x,t) \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1+\text{tg}^2 \delta_\omega(x,t)}}\right)} \times$$
$$\times \left(\text{Exp}\left[-2\int_{-L/2}^{x} \alpha(x,t)\,\text{dx}\right] + \text{Exp}\left[-2\int_{x}^{L/2} \alpha(x,t)\,\text{dx}\right]\right),$$

where $\text{Re}\left[\sqrt{\frac{\tilde{\bar{n}}_\omega^{\text{eff}}(x,t)}{\bar{n}_\omega^{\text{eff}}(x,t)}}\right] = \sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{1+\text{tg}^2\delta_\omega(x,t)}}\right)}$ is the same equal notation.

Given the ratio $k_0 \bar{n}_\omega^{\text{eff}}(x,t) = \beta(x,t) - i\alpha(x,t)$, where $\alpha(x,t) = -k_0 \text{Im}\left[\sqrt{\bar{n}_\omega^{\text{eff}}(x,t)}\right]$ and $\beta(x,t) = k_0 \text{Re}\left[\sqrt{\bar{n}_\omega^{\text{eff}}(x,t)}\right]$ are the indicators of adsorption and transmission of plane electromagnetic wave, we obtain the effective quantities of wave length $\lambda_\omega^{\text{eff}}(x,t)$ and penetration depth $D_p^{\text{eff}}(x,t)$ for such wave process

$$\lambda_\omega^{\text{eff}}(x,t) = \lambda_0 \frac{\sqrt{2}}{\sqrt{\bar{\varepsilon}_\omega^{\text{eff}(1)}}} \frac{1}{\sqrt{\sqrt{1+\text{tg}^2\delta_\omega(x,t)}+1}},\ D_p^{\text{eff}}(x,t) = \frac{c_0}{\omega} \frac{\sqrt{2}}{\sqrt{\bar{\varepsilon}_\omega^{\text{eff}(2)}}} \frac{1}{\sqrt{\sqrt{1+\text{tg}^2\delta_\omega(x,t)}-1}}$$

under conditions of executing of this $2\alpha\beta = k_0^2 \sqrt[3]{\text{tg}^2\delta_\omega} \text{Re}\left[\bar{\varepsilon}_\omega^{\text{eff}}\right]$ rationed relation, where $\text{tg}\delta_\omega(x,t) = \bar{\varepsilon}_\omega^{\text{eff}(2)}/\bar{\varepsilon}_\omega^{\text{eff}(1)}$ is the dielectric loss factor.

Thermal stress under moisture loads for a spatially isotropic homogeneous solid phase (framework or skeleton) in a porous medium under the condition of the absence of external force loads, the quasi-static problem of linear thermoelectricity is formulated in the form of the ratios

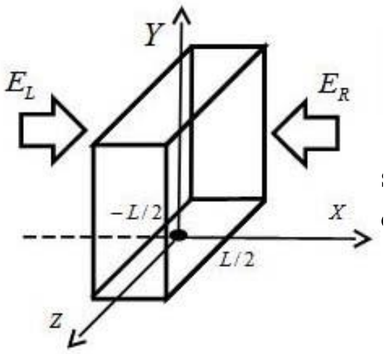

Figure 7: Shematic Depiction of Porous Plate Under Microwave

$$\hat{\mathbf{T}} \quad \hat{I}F \quad , \quad -\langle\varphi\rangle\left[— \quad \eta_L\frac{\partial P_C}{\partial x_\alpha}\right]+\langle\varphi\rangle P_C\frac{\partial \eta_L}{\partial x_\alpha} \quad ,$$

where $\hat{\mathbf{T}}^* = (1-\langle\varphi\rangle)(\hat{\mathbf{t}}'_s - \langle K\rangle\langle\alpha_S^T\rangle(T-T_0)\hat{I})$ is the leading stress tensor, F_α is the components of internal (ponder motore) forces due to gas pressure P_G, capillary pressure P_c and liquid pore saturation η_L under averaged constant porosity of body $\langle\varphi\rangle$ and $\hat{I}$ is a diagonal tensor.

In the case of a one-dimensional plate (Fig. 7), which is under influence of internal microwave heating according to conditions of compatibility of Saint-Venant deformations, the system of equations for the stress component of the tensor has the following form

$$—— + \frac{}{\partial x} = 0$$

$$\frac{\partial \quad {}_{\mathrm{yy}}}{\partial x^2} \quad \frac{}{1-\langle\nu\rangle}\left(\langle E\rangle\frac{\partial^2}{\partial x^2}+\langle\nu\rangle\frac{\partial F}{\partial x}\right) \quad \frac{\partial^2 T^*_{\mathrm{zz}}}{\partial x} \quad \frac{1}{1-\langle\nu\rangle}\left(\langle E\rangle\frac{\partial^2 L}{\partial x^2}+\langle\nu\rangle\frac{\partial F}{\partial x}\right)$$

Force loads in the internal volume of the plate are determined as

$$\boldsymbol{\Phi}^*(x) = -\int_0^x F(x)\mathrm{dx} = -[\boldsymbol{\Phi}(x)-\boldsymbol{\Phi}(0)], \quad L^*(x) = L(x) - L(0), \qquad (9)$$

where $\boldsymbol{\Phi}(x) = -\int F(x)\mathrm{dx} = \langle\varphi\rangle[P_G - \eta_L \mathrm{P}_C]$ and $L(x) \equiv -(1-\langle\varphi\rangle)\langle K\rangle\langle\alpha_S^T\rangle(T-T_0)$ are the quantity of humidity and thermal stress and their corresponding constant values $\boldsymbol{\Phi}(0)$ and $L(0)$ on the internal surface of symmetry (see Fig.2 is the surface YZ at $x = 0$) for the porous plate [39].

According to the symmetry conditions, the boundary conditions for a plate of finite L thickness are applied: 1. There are no internal forces $\boldsymbol{\Phi}^*(0) = L^*(0) = 0$ along the plane of symmetry of a plate; 2. The boundary surfaces of the plate are free from the external loads $\int_{-L/2}^{L/2} T^*_{\mathrm{xx}}\mathrm{dx} = \int_{-L/2}^{L/2} T^*_{\mathrm{yy}}\mathrm{dx} = \int_{-L/2}^{L/2} T^*_{\mathrm{zz}}\mathrm{dx} = 0$; 3. The derivative of stresses along a plane of symmetry $\frac{\partial T_{\mathrm{xx}}(0)}{\partial x} \quad \frac{\partial T^*_{\mathrm{yy}}(0)}{} = \frac{\partial T^*_{\mathrm{zz}}(0)}{} \quad 0$ is equal to zero.

Then the solution of the equations of the quasi linear thermoelastic problem under the action of internal (moisture and thermal) loads has the form of stresses

$$= \boldsymbol{\Phi}^*(x) - \langle\boldsymbol{\Phi}^*(x)\rangle, \quad T^* \ (x) = T^* \ (x) = \frac{\langle E\rangle}{-\langle\nu\rangle}\left[M^*(x) - \langle M^*(x)\rangle\right]$$

and deformation

$$E^*_{\mathrm{yy}}(x) = E^*_{\mathrm{zz}}(x) = \frac{1}{\langle E\rangle}\left[(1-\langle\nu\rangle)T^*_{\mathrm{yy}}(x) \quad \langle\nu\rangle T \quad (x)\right] \quad L(x) \quad L(0),$$

$$E^*_{\mathrm{xx}} \quad \frac{1}{\langle E\rangle}\left[T_{\mathrm{zz}} - 2\langle\nu\rangle T^*_{\mathrm{yy}}\right] + L(x) - L(0),$$

correspondingly, where $M^*(x) = L^*(x) - \frac{\langle\nu\rangle}{\langle E\rangle} \quad ^*(x)$ is an expression for the total internal forces, and $\langle M^*\rangle = \frac{1}{L}\int_{-L/2}^{L/2} M^*(x)\mathrm{dx}$ and $\langle \Phi^*\rangle = \frac{1}{L}\int_{-L/2}^{L/2} \quad (x)\mathrm{dx}$ are the corresponding averaged effort values.

1.4. The main mechanical the rmodiffusion equations

In such way, we obtain the system of closed equations of heat and moisture diffusion

$$(\rho C_p)_{\mathrm{eff}}\partial_t T(x,t) - \frac{1}{2}\langle r\rangle\partial_t[\langle c_L\rangle^L - \partial_t\langle c_v\rangle^G] = \partial_x[\lambda_{\mathrm{eff}}\partial_x T(x,t)] +$$

$$+\frac{1}{}\langle r\rangle\vec{\nabla}\cdot\ [\langle J^x\rangle - \langle J^x_v\rangle] \quad \dot{q}^t(x),$$

$$\partial_t\langle c_L\rangle^G + \partial_x\langle\vec{J}_L\rangle = \dot{I}\ ,\ \partial_t\langle c_v\rangle \qquad \langle\vec{J}_v\rangle \quad -\dot{I}$$
$$\partial_t\langle c_a\rangle^G = -\partial_x\langle J^x_a\rangle$$

with the corresponding boundary conditions described above, the solutions of which can be found according to the obtained expression for the heat sources

$$\dot{q}^t(x) = 2P_{\mathrm{in}}|\bar{n}_\omega(t)||T(t)|^2\alpha(x,t)\sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{\qquad (x,t)}}\right)}$$
$$\times\left(\mathrm{Exp}\left[-2\int_{-L/2}^{x}\alpha(x,t)\,\mathrm{dx}\right] + \mathrm{Exp}\left[-2\int_{x}^{L/2}\alpha(x,t)\,\mathrm{dx}\right]\right)$$

and may determine the stress-strain state of the body with values the calculated relative to the plane of symmetry

$$\Phi(x) = -\int F(x)\mathrm{dx} \quad \langle\varphi\rangle[P_G(x,t) \qquad _L\ _C(x,t)]\,L(x) \quad -(1 \quad \langle\varphi\rangle)\langle K\rangle\langle\alpha_S\rangle[T(x,t) \qquad _0]$$

humidity and thermal forces loadings.

The closed system of equations is obtained by fulfilling the conditions of weak variability of volume (phase), dielectric (wave) properties of an investigated three-phase porous wetted body

$$\frac{1}{\theta_\sigma(x,t)}\frac{\partial\theta_\sigma(x,t)}{\partial t} << \omega_0 \quad \text{and} \quad \frac{1}{\bar{n}^{\mathrm{eff}}_\omega(x,t)}\frac{\partial\bar{n}^{\mathrm{eff}}_\omega(x,t)}{\partial x} << k^{\mathrm{eff}}_\omega(x,t)$$

as well as conditions

$$\lambda^{\mathrm{eff}}_\omega(x,t) = \frac{2\pi\upsilon^{\mathrm{eff}}_\omega(x,t)}{\omega_0} >> l,$$

which are determines the possibility of applying the approximation of the effective macroscopic field in the study (determination) of the effective electro-physical properties of the porous body according to the method of local spatial averaging.

Here $k^{\mathrm{eff}}_\omega(x,t) = 2\pi/\lambda^{\mathrm{eff}}_\omega(x,t)$ and $v^{\mathrm{eff}}_\omega(x,t) = c_0/\bar{n}^{\mathrm{eff}}_\omega(x,t)$ wave vector and phase velocity of electromagnetic (T.E.M) wave propagation in the simulated environment, $\bar{n}^{\mathrm{eff}}_\omega(x,t)$ is the effective value of refractive index, $\theta_\sigma(x,t)$ is the volume fraction of σ- phase (here ω_0 is the angle frequency and l is the characteristic (R.E.V) length of averaged volume).

2. Analysis and Discussion

Numerical simulation of the stress state of a flat unbounded plate under symmetric microwave irradiation was carried out for the selected porous material: historic ceramic brick. The sorption and capillary properties of the studied material are described in detail into the paper [40]. In particular, according to a comparative analysis of semi-empirical models of moisture release or retention, it was established, then at defined averaged porosity $\langle\varphi\rangle = \mathbf{0.46}$ the internal permeability of porous skeleton is $k_{\text{in}} = 2.29\cdot10^{-13}\ m^2$ and empirical parameter in the model of relative permeabilities for fluid (liquid and gas) phase by van Genuchen [41] $m = 0.67$. Also into the work the relative permeability of liquid k_{rL} and gas k_{rG} phases are defined [40]. Its pointed, that capillary properties of porous material are identically determined by the use of dimensionless J- Leverett function, which takes a form

$$J(\eta_L) = \left([1/\Theta\,(\eta_L)]^{1/} \qquad 1\right)^{1-m},$$

here $\Theta = (\eta_L - \eta_L^{\text{ir}})/(\eta_L^{\text{cr}} - \eta_L^{\text{ir}})$ is effective pore saturation by liquid, $\eta_L^{\text{ir}} = 0.015$ and $\eta_L^{\text{cr}} = 1$ are residual and critical pore saturation by liquid for the mentioned material accordingly.

The beginning or initial equilibrium wetting η_L^{eqv} of porous material is defined by the known relation

$$\eta_L^{\text{eqv}} = \left(\eta_L^{\text{cr}} - \eta_L^{\text{ir}}\right)\Big/\left[1 + \left(\frac{P_{\text{amb}}\sqrt{k_{\text{in}}/\langle\varphi\rangle}}{\sigma(T_{\text{amb}})}\right)^{\frac{1}{1-m}}\right]^m + \eta_L^{\text{ir}}, \qquad (10)$$

where $P_{\text{amb}} = 10325[\text{Pa}]$ and $T_{\text{amb}} = 293.15[K]$ are pressure and temperature of the vapour like mixture in surrounding equipment accordingly, and is coefficient of surface tension [40].

At calculation of matrix elements in the generalized form of heat and mass equation according to known expressions (8) for phase and component flows the following values of molar mass $M_a = 0.029[\text{кmol/kg}]$ for dry air and $M_v = 0.018[\text{кmol/kg}]$ water vapour in two component gas mixture are used. Then the expression for density $\langle\rho_G\rangle^G$ of gas mixture in the approximation of light solution $P_G = p_v/x_v$ has been view

$$\langle\rho_G\rangle^G = \frac{P_G\,M_G}{RT} = \frac{M_v + [(1-x_v)/x_v]M}{RT} = \langle\rho_v\rangle^G + \langle\rho_a\rangle^G$$

where $p_v = \phi p_{vs}(T)$ a is partial pressure of unsaturated water vapour, ϕ is relative humidity of air, p_{vs} is the equilibrium pressure of saturated water vapour, for this value a temperature approximation is satisfied

$$p_{vs}(T) = \text{Exp}\begin{bmatrix}-5800.2206/T + 1.3914993 - 4.8640239\cdot10^{-2}T + +4.1764768\cdot10^{-5}T \;- \\ -1.4452093\cdot10^{-8}T^3 + 6.5459673\text{Ln}(T)\end{bmatrix},$$

here T is thermodynamic temperature [42].

Because relative humidity ϕ is defined according to the known for investigated material dependence of the capillary pressure $p_c(\eta_L, T) = \sigma(T)\sqrt{\langle\varphi\rangle/k_{\text{in}}}\,J(\eta_L)$ from the temperature T and the pore saturation η_L by the liquid, in such case the equilibrium (initial) value of water vapour molar fraction x_v^{eqv} in wetted porous material is

$$x_v^{eqv} = \phi_{eqv}\,p_{vs}(T_{amb})/P_{amb}$$

where $\phi_{eqv} = Exp[-M_v\,p_c(\eta_L^{eqv}, T_{amb})/\langle\rho_L\rangle^L RT]$ is the initial value of relative humidity, η_L^{eqv} is the corresponding equilibrium (beginning) value of wetting, which has been defined according to relation (10) above.

Effective thermal characteristics of the composite material have been calculated according to known experimental propertied of phases, as it depicted in the Table 1 [43,44].

Phases	Solid (S)	Liquid (L)	Gas (G)
Thermal conductivity $[Wm^{-1}K^{-1}]$	0.55	0.65	0.025
Thermal capacity $[J\text{kg}^{-1}K^{-1}]$	810	4180	1006

Table 1: The Thermal Properties of Phases in the Wetted Porous Material

For the defined effective $\bar{\varepsilon}_{\omega}^{\text{eff}}$ electro physical properties (11) in the investigated material, including dielectric loss tangent $\mathbf{tg}\boldsymbol{\delta}_{\omega}$, the numerical calculations were performed in the approximation of the effective medium (E.M.A), according to the relation

$$\sum_{\sigma} \theta_{\sigma} \frac{\varepsilon_c^{\sigma}(\omega)-\bar{\varepsilon}_{\omega}^{\text{eff}}}{\varepsilon_c^{\sigma}(\omega)+2\bar{\varepsilon}_{\omega}^{\text{eff}}} = 0. \quad (11)$$

The last cubical equation is solved relatively to effective dielectric constant of composite $\bar{\varepsilon}_{\omega}^{\text{eff}}$, where coefficients are defined by known dielectric constant $\varepsilon_c^{\sigma}(\omega)$ and volume fraction θ_{σ} of porous media component with usage of experimental data, as it is depicted into Table 2 [45].

ν, [Gz]	ε_S		ε_L		ε_G	
	ε_S'	ε_S''	ε_L'	ε_L''	ε_G'	ε_G''
$2.45 \cdot 10^9$	5,86	0,703	80	20	1	0

Table 2: Dielectric Constants of Components for Investigation Material According to E.M.A Approximation ($\mathcal{E}_S$- solid phase, $\mathcal{E}_L$- water, $\mathcal{E}_G$- air)

At calculation, it is taken into account, that density of solid skeleton $\langle\rho_S\rangle^S = 1500[\text{кg}/m^3]$ and incompressible liquid $\langle\rho_L\rangle^L = 1000[\text{кg}/m^3]$.

A surface tension coefficient σ and latent heat (enthalpy) of the vaporization $\langle r\rangle$ [46] has been reviewed in the form of approximated temperature dependencies

$$\sigma(T) = 0.121978 - 0.0001683T$$

and

$$\langle r\rangle = 2.672 \cdot 10^5 (T - T_{cr1})^{0.38},$$

where $T_{\text{cr1}} = 647.3[K]$ is the critical temperature (triple point).

The dynamical viscosity of binary gas mixture has been modelled according to Wilke theory, when for clear components of water vapour μ_v and dry air μ_a it is selected the following numerical approximations

$$\mu_v(T) = 8.85 \cdot 10^{-6} + 3.53 \cdot 10^{-8}(T - T_{\text{cr2}}),$$
$$\mu_a(T) = 17.17 \cdot 10^{-6} + 4.73 \cdot 10^{-8}(T - T_{\text{cr2}}) + 2.22 \cdot 10^{-11}(T - T_{\text{cr2}})^2$$

here $T_{\text{cr2}} = 273.15[K]$ is the fixed (critical) temperature of water freezing, also

$$\mu_L(T) = 0.6612 \cdot 10^{-8}(T - 229)^{-1.562},$$

where μ_L is the dynamical viscosity approximation according to data [47-49].

Diffusion properties of components of vapour-air mixture in the pores have been defined with usage of introduced by the extended through the author of research the diffusion coefficient

$$\text{eff} \quad - \frac{\langle\varphi\rangle(1 \quad _L)}{-\langle\varphi\rangle(1 \quad _L)(x \quad 1)} \quad ,$$

where D $.29 \cdot 10$ $^{13}[m/s]$ is the diffusion coefficient of water vapour into the dry air.

3. Conclusions

According to results of heat and mass process numerical simulation the time shift of near surface wetted and heated zones which are separated by the surfaces S_1 and S_2 is obtained due to distribution of dimensionless sickness $\Delta h_1/L$ and $\Delta h_2/L$ as it depicted on the (Fig.8) and (Fig.9) relatively to surface S_0 of plate symmetry.

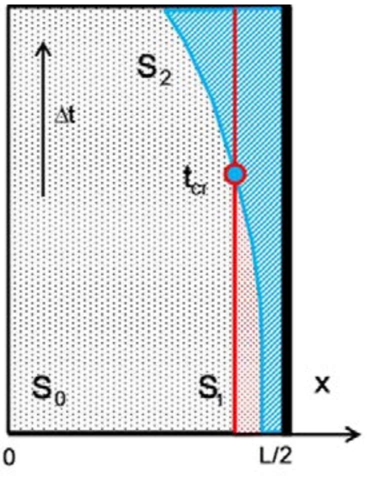

Figure 8: The dynamics of change of near surface heated and wetted zones (schematic representation)

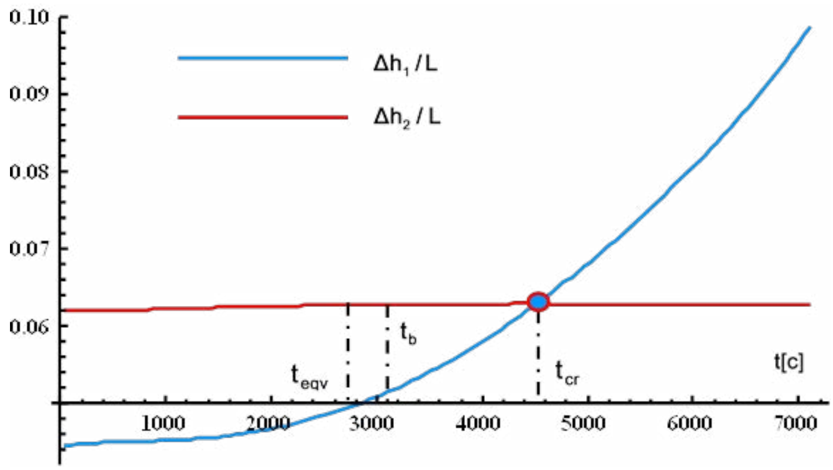

Figure 9: The dynamics of change of near surface heated and wetted zones (graphical representation)

Conclusion I: 1. It is existing a fixed critical value of a time t_{cr} there is a point of the crossover of surfaces S_1 and S_2 about that the surface mass transfer coefficient h_m^* (Fig.10) increase non lineally. It can be joined with beginning of the processes of intensity liquid evaporation at the near surface wetted zone;

2. It is also not significant the nonlinear behaviour of surface heat transfer coefficient is delivered h_T^* (Fig.10) along the surface S_1, which is produced with deviation of the averaged liquid pore saturation η_L from an equilibrium values at surface of symmetry S_0 of plate.

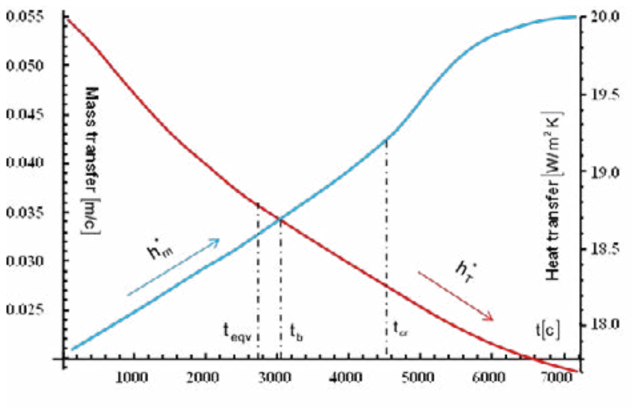

Figure 10: The surface emission coefficient for heat and mass transfer

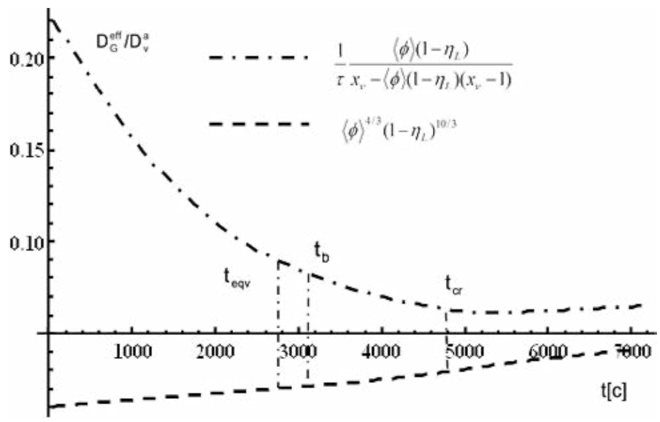

Figure 11: The values of internal and surface diffusion coefficients at the evaporation surface

Conclusion II: 1. In the neighbourhood of time value t_{eqv} an equilibrium state of two component mixture of water vapour and dry air is fixed ($x_v = x_a \simeq 1/2$), above it the dry air is gradually displaced through the surface S_1 of wetted and molar fraction of water vapour x_v in critical point of time t_{cr} follows to maximum value $x_v = 1$ of saturation. 2. Diffusivity D_G^{eff}/D_v^a of gas mixture in pores of material for the internal volume of the plate and at the evaporation surface are defined in various way, but over time in the region $t \geq t_{cr}$ the calculated values (at $\tau = 4$ this model tortuosity factor) of diffusion coefficients on the surface S_1 is aligned.

Solutions of modified system of heat and mass balance for material of historical ceramic brick are represented in the form of graphical dependencies as it's showing below, here $L^* = n^* \Delta L/L_0$ $(n^* \quad \Delta L/L_0)$ is dimensionless half thickness of plate.

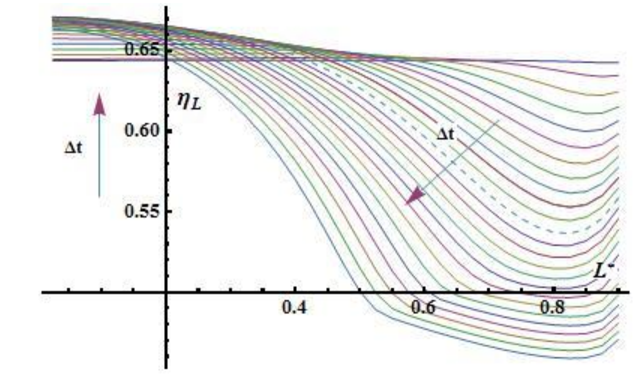

Figure 12: The distribution of the pore saturation by liquid

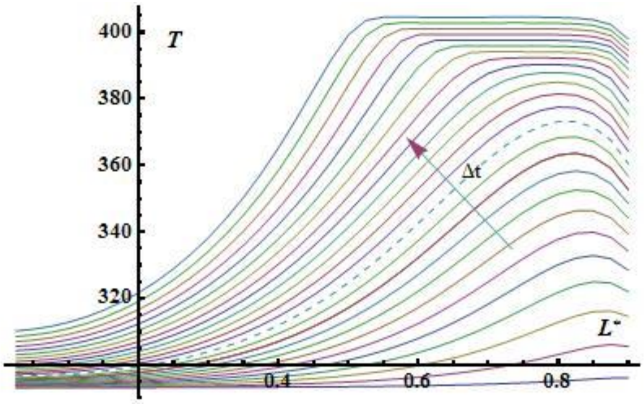

Figure 13: The distribution of the temperature field

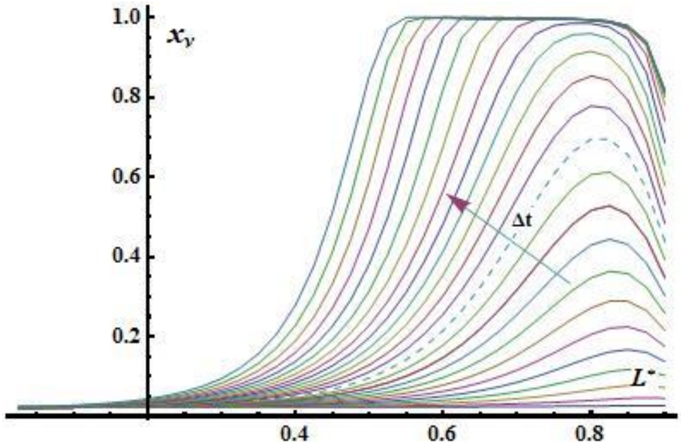

Figure 14: The distribution of the molar fraction for water vapour

Conclusion III: Decreasing of moisture content is caused by increasing of evaporation sources that as a result of inhomogeneous microwave heating of plate and change of thermodynamic state of two component mixture.

On the pictures below its depicted the distribution of flows by liquid (Fig.15), water vapour (Fig.16) and dry air (Fig.09) along half thickness of plate in the different time intervals.

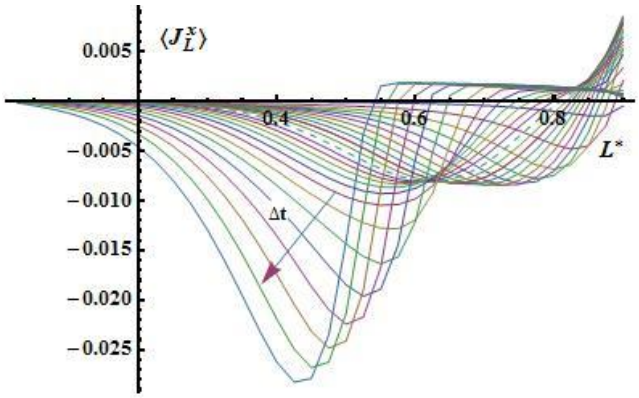

Figure 15: Distribution of the liquid flow

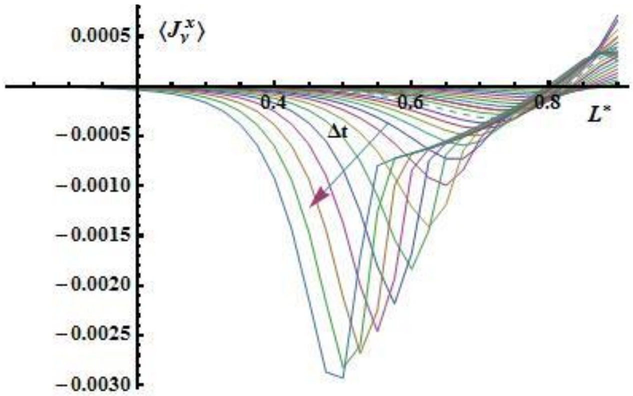

Figure 16: Distribution of the water vapour flow

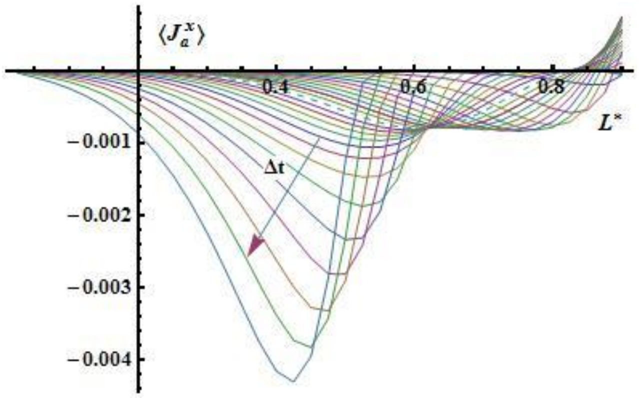

Figure 17: Distribution of the dry air flow

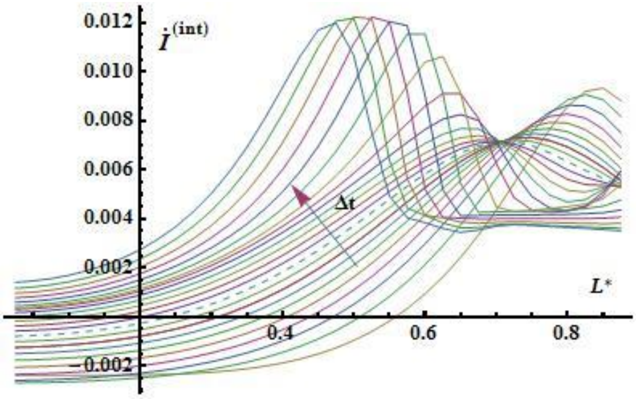

Figure 18: Distribution of an internal evaporation sources

Conclusion IV: In the neighbourhood of time point t_b, which corresponds to achievement of boiling temperature at normal conditions, its fixed change of flows distribution for liquid and gas mixture components by thickness of plate (dashed line). This is due to the increase in the intensity (Fig. 18) of internal sources of evaporation $\dot{I}^{(\text{int})}$ along the thickness of the plate.

The distribution of specific stresses (Fig. 19) and (Fig. 20) and small deformations (Fig. 21) and (Fig. 22) in a solid matrix (frame or skeleton) over the half-thickness of the plate is shown by the following graphical dependencies.

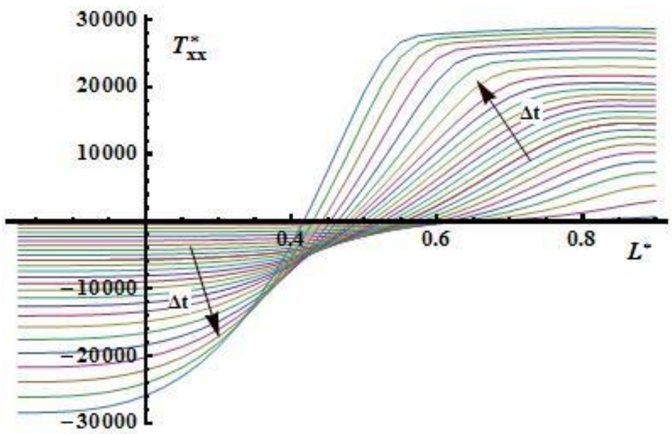

Figure 19: Distribution of stress in longitudinal direction

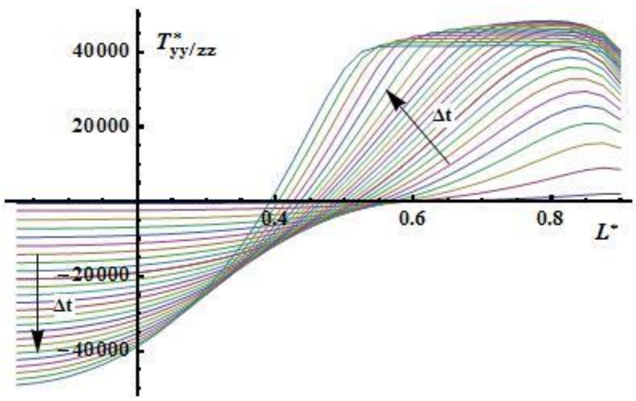

Figure 20: Distribution of stress in tangential direction

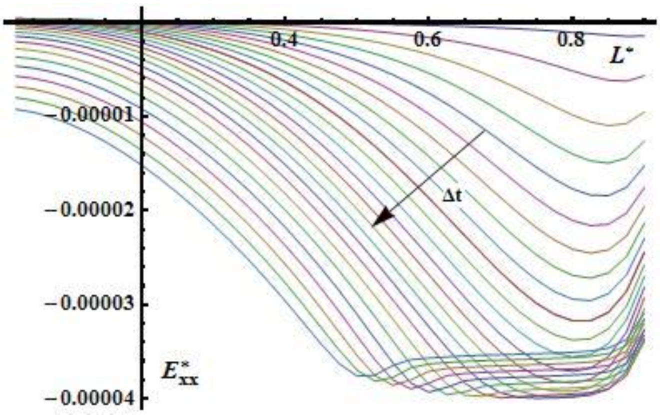

Figure 21: Distribution of strains in longitudinal direction

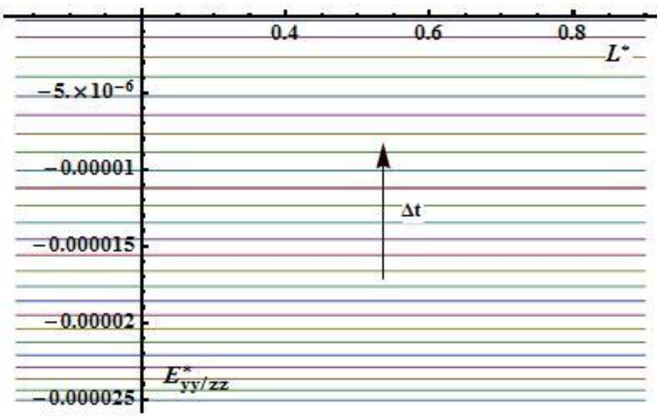

Figure 22: Distribution of strains in tangential direction

The dynamics of thermal stress distribution (Fig. 23) along the thickness of the plate is also shown in the form of the corresponding graphical dependence:

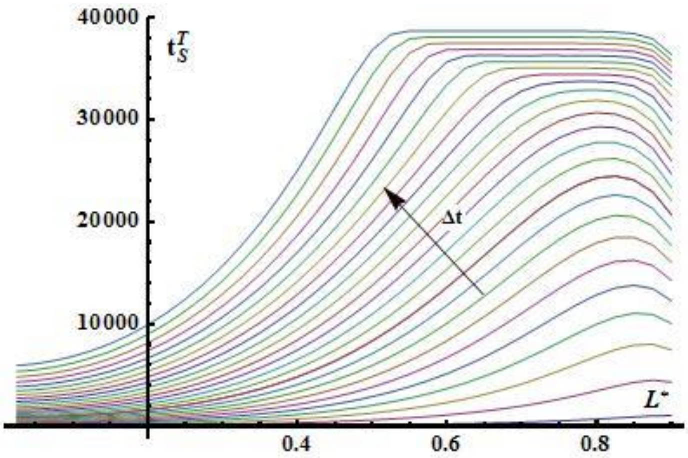

Figure 23: Distribution of thermal stress in porous skeleton

From the pictures below (Fig.19 and Fig.20) it is shown that longitudinal components of stress tension is joined first with distribution of the temperature field (see Fig.8) and nonlinear behaviour of tangential component of stress tension is results of influence of nonlinear evaporation processes (see Fig.12) especially near surface of sample.

Acknowledgment

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