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Explorations in Astronomy & Astrophysics

Review Article

The Lorentz Transformation, Time and Space. Generalization of the γ Factor as a Function of the Direction of the Hidden Movement in the Clocks

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Received: 2025 July 28 **Accepted:** 2025 Aug 06 **Published:** 2025 Aug 28

Abstract

Our purpose is to examine the articulation between the Lorentz transformation (theory of relativity) and the very construction of the concept of time. The latter does not exist ready, waiting to be enrolled in the equations of mathematical physics. However, when deriving the transformation, the existence of time is not contested. Under these conditions, the notion of a clock is not the subject of any special discussion. We want to criticize this classic approach because, if we want to deepen and contest this very existence of time, we are led to «open» the clocks and look at what happens there... We realize that we always transform a movement into time (a clock is a point of view on a movement). In these conditions, it is a question of taking up in a concrete way, i. e. by looking at this or that particular clock, what is happening about time. In the case of an atomic clock like today, it is a movement of light. In this context, we are led to compare the relation of the same photon to two reference frames (at rest / mobile; this is how we define the two clocks) while assigning it the same speed (second postulate of relativity). The new approach leads to the study the writing of the transformation according to the orientation of the movement defining time. The meaning of the γ relativity factor is extended, making it depend not only, as in the standard case, on the ratio $\beta = v/c$ of the velocity modules (the relative movement of the reference frames and light), but also on the angle δ between these two movements. A general relation $\gamma = \gamma(\beta, \delta)$ is proposed which makes it possible to find in the same framework various transformations already known, including Lorentz's (the intelligence of the word transformation is renewed in the sense of accommodating a particular movement of light, and not all possible movements). This approach provides research perspectives on certain problems in astronomy and astrophysics.

Keywords: Lorentz Transformation, Clock, Space, Time, Movement, γ Factor, Factor $\beta = v/c$, Angle δ, Light, Atomic Clock, Optical Clock, Reference Frame At Rest, Moving Reference Frame, Second Postulate, Relativity Theory

1. Introduction

In relativity theory, the Lorentz transformation makes it possible to move from a collection of space and time variables (x, y, z, t), evaluated in a reference frame R said at rest, to another collection (x', y', z', t') evaluated in a moving reference frame R' at a speed v with respect to the first. We will use the word speed (with or without quotation marks) here, although for light it is not appropriate (see the discussions in the literature and in our work). Our purpose here is to examine the articulation between this transformation and the construction of the concept of time. The latter does not exist ready, waiting to be enrolled in the equations of mathematical physics. However, when deriving the transformation, the existence of time is not contested: it is then a question of establishing its measurement and comparing the estimated values in the two reference frames in relative motion, under various constraints, in particular that of an equal "speed" c of light in each reference frame (the consequence is the relativity of time). In these circumstances, the notion of a clock is not the subject of any special discussion: it is assumed that it is available as much as necessary. Located at the various locations of the reference frames, the clocks give the local time; they are even assumed to be punctual for the proper functioning of the reasoning (after the so-called synchronization process, we can limit ourselves to one clock per reference frame).

We want to criticize this classic approach because, if we want to dig a little deeper and challenge this very existence of time, we are led to "open" the clocks and look at what is happening there... We realize that we always transform a movement into time (a clock is a point of view on a movement). In the case of an atomic clock (also known as an optical clock) used as a reference as today, it is a movement of light that is involved. The question then arises as to how this movement of light is or is not in harmony with the movement of "external" light that we have just discussed.

The answer assumes that the same choices are made for the two or three (including the two clocks associated with the two reference frames) lights, which are now only one from a conceptual point of view. Indeed, we are logically led to make the same assumption "c = cte" for the two internal photons of the two clocks as for the external photon. As a result, all three photons play the same role and only one photon is required. The same photon serves both as a clock, or as plural clocks (each point of view on it is a clock; we have one for each reference frame) and as support for the postulate "c = cte". There is no other photon to consider, nor is there any time that would flow locally here or there, to measure here or there.

Because of the relative displacement between the two reference frames, the direction of the movements will then intervene: the movement in R does not have the same orientation as in R'. Previously, there was no interest in the direction of the movement for two additional reasons: - we did not open the clocks: a scalar time came out of them as if by magic; - we were interested in all possible directions for the "external" photon (whose "constant" speed was taken). The orientation of the relative displacement between the two reference frames was relevant for the offset of the origins, but not in the γ distortion factor.

It is ultimately the understanding in terms of movement of time itself that presents itself to us in Lorentz transformation. There is no longer time on one side (hidden in the clock) and light on the other, it is the same thing; time being marked by the movement of light. This statement ipso facto particularizes the direction of light in the "real" "clock", and asks us not to accommodate all possible movements (a summary of our approach is given in Figure 1).

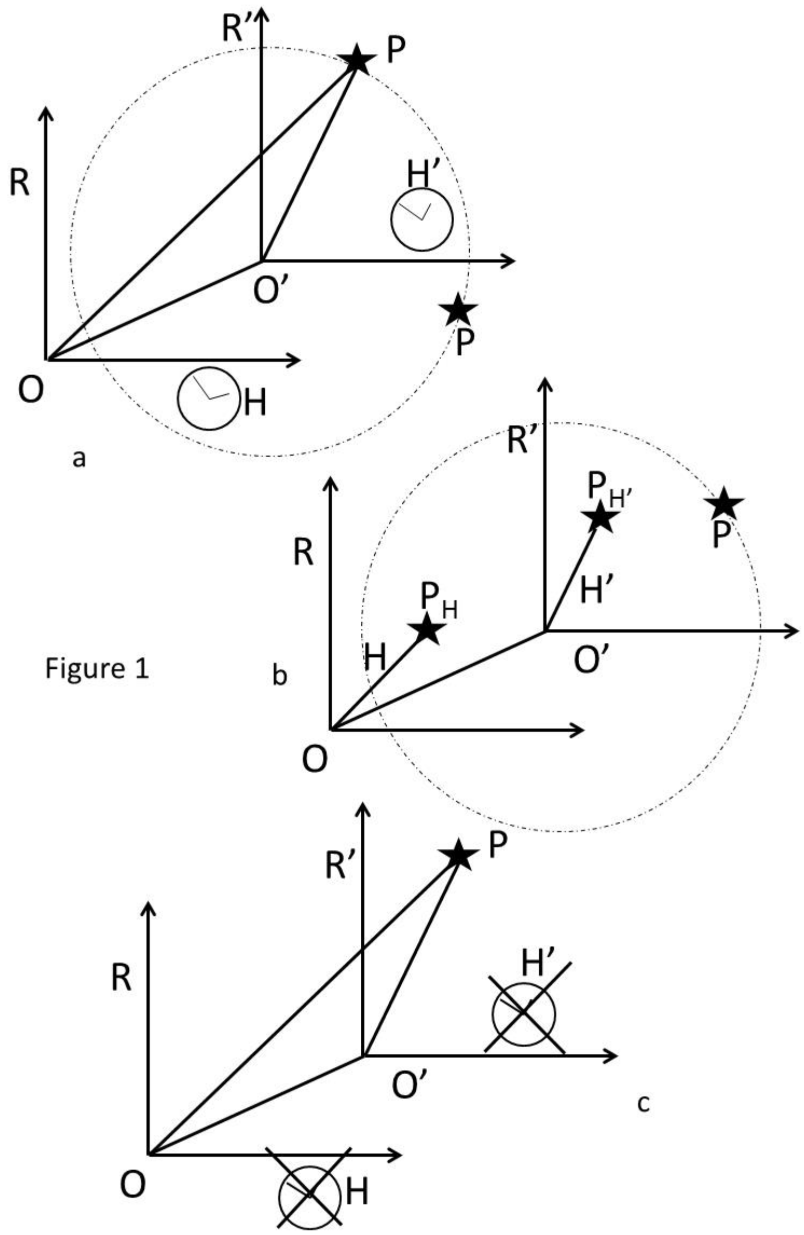

Figure 1: Illustration of the Steps of our Proposal

In 1a, the situation described by the classical approach is represented. Two reference frames R and R' of origin O and O' are relatively moving to the speed v (OO' = vt). Each of the two reference frames is equipped with a clock, H and H'. We consider the displacement of a photon P and assume that its displacement speed is the same viewed from R and from R'. This photon can move in all directions: this possibility has been represented by the wave surface centered on O', i.e. a two-dimensional circle (on which two photons P have been pictured, we could put as many as we want).

In 1b, the clocks H and H' were opened and we found that each uses the movement of a photon, PH and PH'. We now have three photons, PH, PH' and P, counting the latter as an infinity in the different directions.

In 1c, the approach was unified, being led to postulate the same behavior for the different photons that are now one at the conceptual level. To have clocks that are needed to "measure time" in both reference frames, we are thus led to particularize a photon that will serve both for time in R and in R' (while "embodying" the second postulate of relativity). The two clocks H and H' as defined in Figure 1a are destroyed (or forgotten, they were fictions).

In total, this proposal makes the relativistic approach coherent: one could wonder (within the understanding 1a) by what mystery the clocks offered a time that shifted from one reference frame to another, to satisfy the properties of a photon whose journey was external to them, a time that was variable with the relative movement of the reference frames. The inconsistencies of the relativistic approach have been analyzed by critical physicists, but, in our opinion, without any new proposal.

In the context of today's optical clocks, expressing the identity of the properties of light propagation in the two reference frames amounts to considering two points of view (that of R and that of R') on a photon to which the same velocity is assigned, in a reasoning where the implicit geometric content of the equations is as important as the explicit algebraic content. The new approach therefore leads to the study of the writing of the transformation as a function of the orientation of the photon motion defining time; various transformations are then possible according to the directions of the axes of the reference frames with respect to the displacements in play (reference frames, light). In doing so, the understanding of the word transformation is renewed in the sense, as we have noticed, of accommodating a singular movement of light and not all imaginable movements.

Here is what our plan will be. We will start with a few brief reminders on the standard Lorentz transformations (closed clocks), section 2. We will then focus our discussion on the γ transformation factor (it is it that retains its originality), by presenting a new formulation: it will take into account a temporal movement of any direction, both in relation to the coordinate axes and the direction of the relative movement of the reference frames (section 3). We will then look at some remarkable special cases (section 4), before making proposals for complete general transformations explaining the offsets in the origins of the reference frames, in addition to the scale factor γ (section 5). We will then be able to discuss some problems related to the composition of non-collinear Lorentz transformations (section 6), before concluding and briefly discussing what this approach brings to the approach to natural phenomena (section 7). Some borrowings from older texts will be highlighted, with the present set providing new formulations and conceptual insights. The reference [1] gives access to many of our works see also [2,3,4,5].

2. Some Reminders on the Lorentz Transformation

There is no question of proposing a panorama, even partial, of everything that has been written about Lorentz transformation. We will simply comment on it quickly through the delicate points already discussed by us: these concern on the one hand the composition of non-collinear transformations, with the related [7]. question of additional rotations [6]. and on the other hand the twins paradox, the relativity of time and the expected time gaps the previous references giving access to an already abundant literature [6,7].

Let us first look at the classic situation where the question of clocks is not yet raised. The Lorentz transformation is first presented for a relative displacement of the reference frames parallel to a coordinate axis. We are brought back to study the transformation of a pair (x, t) in the reference frame at rest into the pair (x', t') in the reference frame moving at the speed v with respect to the first. The other coordinates are not modified (y = y'; z = z'). The constant speed of light constraint, recalled above, is written along the x-axis (if x = ct, ± then x' = ± ct'). The vector v is parallel to the x-axis; we obtain the well-known formulas.

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2) \qquad \text{with} \qquad \gamma = (1 - v^2/c^2)^{-1/2} \qquad (1)$$

2.1. The γ Factor

The previous transformation shows the product of two different terms: - one expresses the offset of the origins between the two reference frames (in the spatial term (x - vt) and its temporal correspondent ($t - vx/c^2$), - the other a distortion of scale in the γ factor; this depends only on the velocity ratio β= v/c. This two-fold separation results from the different constraints imposed on the structure of the equations of relativity (homogeneity, linearity, etc.) and detailed in the original articles and treatises [8-12]. As we have already argued, it is the γ factor, discussed in the next section, that gives the Lorentz transformation its originality.

In the first derivation proposed above, we observe a perfect symmetry between the two variables x and t (x' and t'), already noted by Poincaré and clearly visible by taking c = 1. The corresponding transformations are called special transformations or "boosts". They define a group; i.e. the composition of two transformations with respective velocities v_1 and v_2 is a transformation of the same type, of velocity v_3, a function of v_1 and v_2 (we write $v_3 = v_1 \oplus v_2$).

The next natural step is to extend the previous result to relative movements between any reference frames, i. e. such that the velocity v is not parallel to the coordinate axes. The approach consists in making a first rotation of the rest reference frame to position its x-axis along the speed v. A special Lorentz transformation is then performed as just now. We then return to the initial directions of the reference frames by reversing the previous rotation. It so happens that this process leads to a series of difficulties: in our analysis, they are due to the fact that we no longer have the symmetry between spatial and temporal variables that we had for the couple (x, t) of the special transformation; the group structure is lost. The solution proposed in the literature consists in releasing the constraint of light propagation at a constant speed along a single direction (the x-axis containing the speed v in the boosts) and writing it in all directions; this is done by preserving the quadratic form $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$. In addition to translations, the geometric operations allowed by the constraints then include a priori rotations of the axes. With regard to the composition of several non-collinear Lorentz transformations, we find a mathematical coherence (group structure), adding new rotations (in addition to the previous ones) depending on the speeds of the different displacements to be composed they are called Thomas rotations [6]. This approach, which has been extensively analysed in the literature, raises many questions, particularly about the physical nature of what is written. Our second announced point of view on Lorentz transformation, on the side of the twins paradox, will be discussed below.

3. New Approach. Proposal of a γ Factor Taking into Account the Direction of Light Propagation

We do not repeat our contribution to this discussion, to which the reader is invited to refer. We are not encouraged to retain the various rotations we have just mentioned, insofar as we feel that a resumption of the Lorentz transformation is necessary. This must be done on the occasion of an "opening" of the clocks and the consideration that time is associated with a movement; as we have said, it is now that of a photon in an atomic clock. The position of this photon is described by three space coordinates, t_x, t_y and t_z and the scalar time corresponds, at least momentarily and locally, to the norm $t = (t_x^2 + t_y^2 + t_z^2)^{1/2}$. A slightly different way of understanding the three parameters useful for defining time is to see them as defining the direction of photon motion (see below). We thus limit ourselves to a particular clock, i.e. the movement of light that travels through it (the clock is here the point of view on the standard photon). We can also specify in which reference frame we locate the direction of the standard movement (the orientation in the other reference frame derives from it, taking into account the relative movement). We will consider it here in the moving reference frame R', and will also look at the movement of the photon from the reference frame at rest R, while decreeing it to keep the speed c in the two reference frames (reversing the points of view does not only mean changing v to – v, but changing the reference frame where we start to designate a direction of light - we have done so here in R' ; by reversing the points of view, it is in R that we must designate the direction of the speed of light-see also [7]. In total, it is no longer a question of postulating the conservation of the "speed" of light in all directions. We can continue to use the interval ds^2 by adding constraints; we no longer envisage a multiplicity of movements of light, with a time on its own side, but a single movement to define a time that respects the "logic" of a clock.

3.1. Notations

Let us set the notations that we will use later. We will use, in addition to the vector v of the standard Lorentz transformation, a vector c' for the movement of light in R', corresponding to a vector c in R. The orientation of the velocity v is defined by the direction cosines 4; (the direction cosines α_i define the orientation of a unit vector carried by v and check $\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$) or ratios, α_i such that $v_x = \alpha_x.v$, $v_y = \alpha_y.v$, $v_z = \alpha_z.v$. The coordinates v_x, v_y and v_z make it possible to define the module v (we will only rarely distinguish vector and module by different notations, the context allowing to understand). The two vectors c and c' have the same norm (it is the expression of Einstein's second postulate), but different directions a priori. The corresponding times t and t' are the scalars that scale the progression of the photon along the directions c and c'. The associated t and t' vectors are carried by the c and c' vectors respectively in the two reference frames. The link between them is written

$$\vec{t} = \frac{1}{c}\vec{c}t \qquad \text{and} \qquad \vec{t'} = \frac{1}{c}\vec{c}'t'$$

where it is important for a good understanding to distinguish between vectors $\vec{c}$ and $\vec{t}$ and scalars c and t (vector modules), as well as for $\vec{c'}$ and $\vec{t'}$ with respect to c' = c and t'. The orientation of the vectors c or t, on the one hand, c' or t' on the other hand is defined by the direction cosines ω_i and ω'_i respectively. These ratios check: $t_x = \omega_x.t$, $t_y = \omega_y.t$, $t_z = \omega_z.t$ (rest reference frame); $t'_x = \omega'_x.t'$, $t'_y = \omega'_y.t'$, $t'_z = \omega'_z.t'$ (moving reference frame); or, by multiplying these relations by the constant c, $(ct)_x = ct_x = \omega_x.ct$ (t is a vector in the first term, and its module in the third one) and also for the other components and in both reference frames.

The last relations can still be written $c_x = \omega_x.c$, $c_y = \omega_y.c$, $c_z = \omega_z.c$ for vector c, and similarly with the ratios ω'_x, ω'_y and ω'_z for vector c'. In order to formulate Lorentz transformations for the vector t (t_x, t_y, t_z), in association with the position vector (x, y, z), we will then reason about the vectors t and t' rather than the vectors c and c'.

3.2. Case of Two Dimensions of Space

To simplify the calculations, we will reason in the case of two space dimensions x and y as in Figure 2. In this case, the notations ω, ω' and α denote angles such as: $\omega_x = \cos\omega$, $\omega_y = \sin\omega$; $\alpha_x = \cos\alpha$, $\alpha_y = \sin\alpha$ in the rest reference frame; and $\omega'_x = \cos\omega'$, $\omega'_y = \sin\omega'$ in the moving reference frame. The vector v is seen in R under the angle α and under the same angle α = α' in R' (we will use α for both). Neither the z-direction nor the variables and parameters associated with this direction are involved. It is useful to define the angle δ = ω' - α' between the vectors c' and v (we will not use the equivalent notation δ' = ω'- α'), i.e. between the relative displacement of the reference frames and the movement of the photon marking time in the moving reference frame (we have said that we locate this movement first). We have here two factors expressing the relations between two movements: the ratio β of the two modules, the angle δ between them. The three angles ω, ω', α (α') and the parameter β= v/c are linked together by the relation (A1) : ω = ω' - Arcsin{β sinδ} (see Appendix 1).

3.3. Expression of the Second Postulate for an Optical Clock

It is therefore for the movement of a photon P marking time, another name of a clock, that we will formulate the second postulate of relativity. Looking at the photon from O, time in R is identified by OP = ct; looking at it from O', time in R' is identified by O'P = ct' (relational and not substantial approach). In both cases, the factor c is the same; that is the expression of the second postulate. The offset of the origins is marked by OO' = vt. Refer to Figure 2.

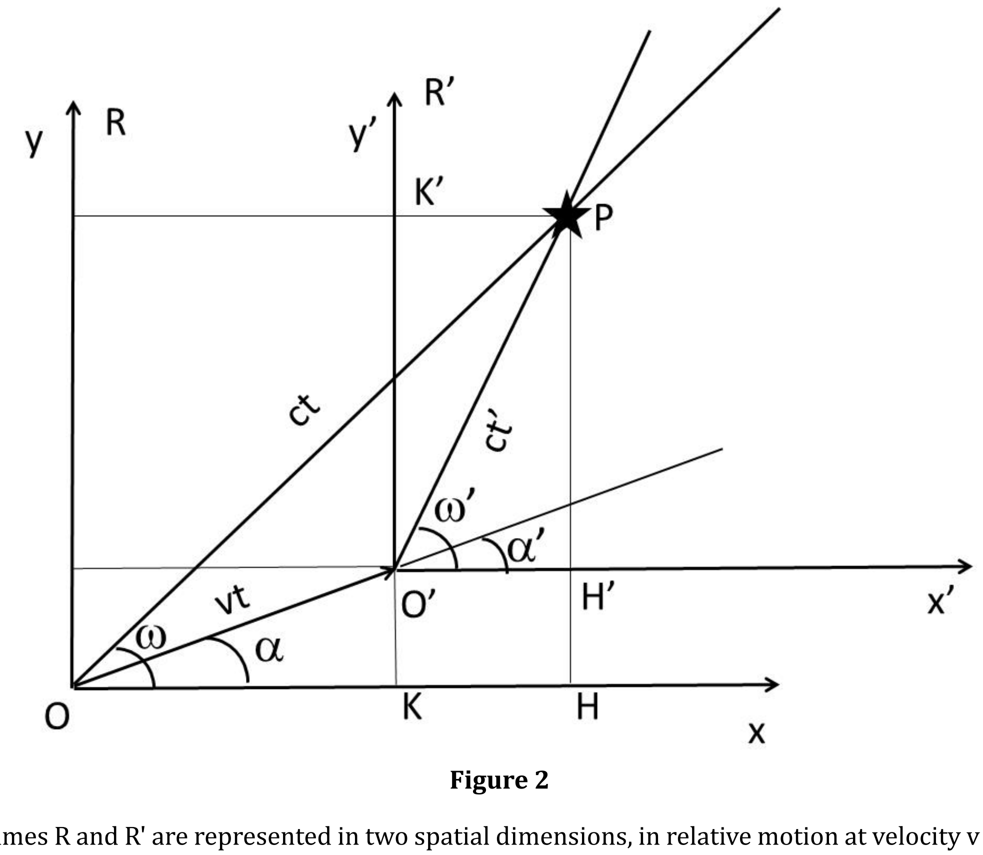

Figure 2

Two reference frames R and R' are represented in two spatial dimensions, in relative motion at velocity v at an angle α to the Ox O'x' direction. In R' the movement of the clock photon P is done in the direction of angle ω'; from the reference frame R, this movement is seen with the angle ω, taking into account the relative movement between the reference frames. During the "time interval" t (measured by a piece of movement), the reference frame R' moved according to OO', the photon P according to O' P in R' and according to OP in R. It is decided / assumed that the photon is seen moving at the same "speed" in both reference frames. We therefore have OP = ct, and O'P = ct', with the same factor c ; the latter being much larger than v, the lengths OP, O'P are much larger than OO' in length vt, which has not been respected for the readability of the figure.

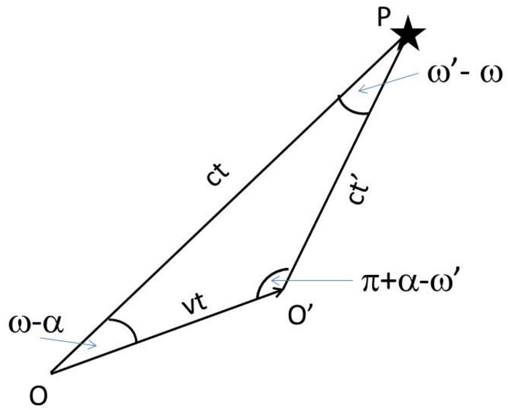

Figure 3: Measurements of the Angles and Sides of the OO'P Triangle

These measurements make it possible to determine the angle ω’ from the angles ω and α, and the ratio β= v/c. The angles of the OO'P triangle are determined from the definitions of α, ω’ and ω given in Figure 2.

By comparing the two times t and t' in the ratio γ (as in classical relativity), we can simply say that the ratio is lower or greater than one. Avoiding a misunderstanding, because of the reflexes acquired by the standard Lorentz transformations ("whatever the interpretation given, the convergence of the two approaches (the classical approach / ours) towards identical equations for particular cases constitutes a certain assurance that we are not getting lost; the functioning of the equations has a meaning in itself") we will not ask ourselves what is seen deformed or not deformed in one reference frame or another, thinking in terms of dilation and contraction of time and length: there is no substantial time or space, waiting to be dilated or contracted; no influence of a movement on the flow of time (which does not exist). We only see movements compared to each other (there are no longer clocks on one side and rulers on the other, positions on one side and times on the other; cf. [13]. Seeking a stable basis for comparison, we are drawn into an infinite regression, which is "temporarily" stopped by simultaneously decreeing the construction of a reference frame and the adoption of a standard signal within it (the photon).

This discussion refers to the debate between Bergson and Einstein on the interpretation of the "flow" of time, and its changes compared between two reference frames (cf. the twins paradox). Without going into detail on the positions of the two scholars (see some elements in [7]. we dare to say that these two authors were both wrong in their belief in a substantial time (which the first considered unique, the second multiple). Bergson was somehow right when he spoke of a parallax or perspective effect to explain the differences expressed by the γ factor; there is indeed a change according to the point of view (OP and O'P are different) but the parallax is original because the effect is not necessarily symmetrical (see above and Guy, op. cit.). And he was wrong in denying any reality at one time in the moving reference frame (he was talking about its fictional, imaginary, fantastical character!): there is something that can be attributed to the R' reference frame. On this point, Einstein was right: the distance of the photon from its origin, i.e. the time in this reference frame, is not illusory (by repeating it in our own words).

3.4. Derivation of the γ Factor

To connect t and t', in the γ factor, the Pythagoras theorem gives us: $OP^2 = OH^2 + HP^2 = (OK + KH)^2 + (PH' + H'H)^2$, what is written :

$$c^2t^2 = (vt\cos\alpha + ct'\cos\omega')^2 + (vt\sin\alpha + ct'\sin\omega')^2$$

$$c^2t^2 = v^2t^2\cos^2\alpha + c^2t'^2\cos^2\omega' + 2vctt'\cos\alpha\cos\omega + v^2t^2\sin^2\alpha + c^2t'^2\sin^2\omega' + 2vctt'\sin\alpha\sin\omega$$

$$c^2t^2 = v^2t^2(\cos^2\alpha + \sin^2\alpha) + c^2t'^2(\cos^2\omega' + \sin^2\omega') + 2vctt'(\cos\alpha\cos\omega' + \sin\alpha\sin\omega')$$

$$c^2t^2 = v^2t^2 + c^2t'^2 + 2vctt'\cos(\omega' - \alpha)$$

$$(c^2 - v^2)t^2 - 2vctt'\cos(\omega' - \alpha) - c^2t'^2 = 0$$

Where traditional trigonometric relations have been used. We obtain an equation of the second degree of t as a function of t' and the parameters v, c, and angle (ω' - α). The reduced discriminant Δ' is equal to :

$$\Delta' = v^2c^2t'^2 \cos^2(\omega' - \alpha) + (c^2 - v^2)c^2t'^2 = c^2t'^2(v^2\cos^2(\omega' - \alpha) + c^2 - v^2)$$

$$= c^2t'^2(c^2 - v^2\sin^2(\omega' - \alpha))$$

This discriminant is always positive and the problem has two solutions. The square root of the discriminant is $ct'(c^2 - v^2 \sin^2(\omega' - \alpha))^{1/2}$ and the solutions are

$$t = \frac{vct' \cos(\omega' - \alpha) \pm ct'\sqrt{c^2 - v^2 sin^2(\omega' - \alpha)}}{c^2 - v^2}$$

That we can write t = γt' with

$$\gamma = \frac{\frac{v}{c}\cos(\omega' - \alpha) \pm \sqrt{1 - \frac{v^2}{c^2} sin^2(\omega' - \alpha)}}{1 - \frac{v^2}{c^2}}$$

Where γ is a function of v, c, and ω' - α. Using the relative notations β = v/c and δ = ω' - α we write :

$$\gamma = \frac{\beta\cos\delta \pm \sqrt{1 - \beta^2 \sin^2\delta}}{1 - \beta^2} \qquad (2)$$

This fundamental relation (2) γ = γ(β, δ) is a function of the ratio of the modules of v (relative movement of the reference frames) and c (movement of light defining time), and of the angle between these movements; γ does not depend on α which will play in terms of the offset between the origins. For the time being, we consider that this is indeed the γ factor of a Lorentz transformation (in a modified sense as explained above), although we have not completely derived it. There are two arguments for this: - the similarity of approach with that derived from the standard transformation (in terms of time "dilation"), - that of the connection with the standard expression (see below). These points may be developed in the future.

3.5. Discussion of the Sign ± in the Expression of γ

What meaning can we give to the choice of sign in the relation (2)? The second term of the numerator and the denominator having a constant positive sign, we must at first sight make a possible change of sign on the first term of the numerator, that is βcosδ. Let us write the factor γ as γ = a ± b. If we take the + sign, we have γ = a + b. If we take the sign -, we have γ = a - b, that we write γ= -(-a + b). In this form, we see that, by keeping the same sign for the second term b of the numerator, we can change the sign, not only of the first factor a, but also of the whole expression of γ. We interpret this as follows: a) the change in sign of the entire expression of γ = t/t' means changing orientation of one of the two t or t'; b) the change in sign of the first term a = βcosδ is obtained either by changing v to - v, or by changing to δ to δ + π (the cosine then changes sign). In total, we will consider that the choice of the sign in the expression of γ refers to questions of orientation of the axes bearing v, t and t', as well as the angle δ. We can choose a sign for one γ from which to find others for other choices of direction.

3.6. Three Dimensional Calculation

Our previous reasoning focused on two dimensions of space x and y. The calculation for three dimensions x, y and z does not present any difficulty in principle. It brings nothing new from the point of view of the fundamental relation (2) that we have demonstrated. Indeed, what is at stake is on the one hand the ratio of the modules of v and c, and on the other hand the angle δ between the vectors c' and v. Since the three vectors v, c and c', or OO', OP and O' P' are *coplanar* by construction (the path of the same photon P is followed), the useful calculation can be done in two dimensions in a plane, even if this plane is immersed in a three-dimensional space. In this case, the value of the angle δ as a function of the direction cosines of vectors c' and v is given in Appendix 2, relation (A2).

4. Study of Some Special Cases (δ = 0, δ = π/2)

We are not going to do a general study of the function γ(δ) to given β (relation (2)) for the moment. A first way to assess the relevance of our fundamental expression is to report specific values of the angle δ.

4.1. Parallel Movement

In the case of a movement of light parallel to the vector v, we have δ = 0, i. e. α = ω', (we will also have ω = ω'); we then have cosδ = 1 and sinδ = 0. These results are also valid for both angles α and ω' simultaneously zero. There remains

$$\gamma = \frac{\beta \pm 1}{1-\beta^2} \quad (3)$$

By taking the + sign, we have

$$\gamma = (1 - \beta)^{-1} = (1 - v/c)^{-1} \quad (4)$$

This is the value already found for parallel transport in Guy corresponding to Elbaz; the formulations given by these two authors contribute to transformations in the renewed sense announced in our introduction [7,14].

If we take the sign -, we have $\gamma = -(1 + \beta)^{-1} = -(1 + v/c)^{-1}$ by changing v to - v and t or t' to - t or - t' in the previous expression (value also proposed by the previous authors).

4.2. Perpendicular Movement

In the case of a movement of light perpendicular to the direction v of the relative movement of the reference frames, we have $\delta = \omega' - \alpha = \pi/2$. Then $\cos\delta = 0$ and $\sin\delta = 1$, and:

$$\gamma = \frac{\pm\sqrt{1-\beta^2}}{1-\beta^2} \quad (5)$$

Or

$$\gamma = \pm (1 - \beta^2)^{-1/2} = \pm (1 - v^2/c^2)^{-1/2} \quad (6)$$

The sign ± referring to the choice of orientation of the t or t' axes. This value corresponds to that given in Guy [7] for perpendicular transport, with or without round trip. We talk about round trip when the motion of the photon is not indefinitely rectilinear but is reflected on a wall, for example. It is also the classic value of Lorentz transformation. One may wonder why the standard transformation corresponds to this geometric configuration. There is then a coincidence, for the formulation of γ, between the choice of a particular path for light (our point of view) and that of accommodating all directions (classical point of view). Various authors have chosen this particular perpendicular path when they want to find a pedagogical way to make the standard Lorentz transformation understood [12,15]. They then identify without saying the "passage of time" with the motion of the photon. One would be tempted to ask these authors why they did not choose parallel transport (which does not give back the standard expression)? In a similar intellectual approach, effective measurements on an optical clock displaced perpendicular to the movement of light give a very interesting return to the standard γ .To our knowledge, this type of experiment has not been done on a parallel trip.

It is precious for us to find the γ factor of Lorentz classical formula, as well as that of Elbaz or our own work, as particular cases of a more general formula where the angle between the directions of the movements of interest of the problem now appears. Table 1 gives various cases of γ factors that we believe can all be found by using the fundamental formula (2). McCarthy's article quoted discusses the general case of physical phenomenon of propagation, measured by clocks based on the same phenomenon of propagation; said brutally, the author's aim is to disqualify the theory of relativity. We will not go that far, the underlying problem of recursivity, posed by all measurements, being inevitable because of our situation inside the world (there is no clock independent of phenomena...): relativity is a way to face it.

Positioning of the movement associated with time with respect to the relative movement of the reference frames Angle value δ	***Lorentz Transformation γ factor*** *With β= v/c*	***Bibliographical references***
Parallel movement in the same direction δ = 0	$(1 - \beta)^{-1}$	Elbaz (14)
Parallel movement in opposite direction δ = 0	$(1 + \beta)^{-1}$	Elbaz (14)
Parallel round trip δ = 0	$(1 - \beta^2)^{-1}$	McCarthy (17)
Parallel round trip Double (x = ± ct, x' = ± ct') δ = 0 All directions (3D)	$(1 - \beta^2)^{-1/2}$	Lorentz (8)
Perpendicular displacement (with or without round trip) δ = π/2	$(1 - \beta^2)^{-1/2}$	Hoffman (12), Rougé (15), Guy (7)

Table 1 : Some Special Cases of the γ Factor Depending on the Angle δ and the Type of Light Movement Chosen (Single, Multiple, with or without Round Trip). The Sign of Each Factor β and γ can be Changed According to the Orientation Choices Discussed in the Text

The γ factors can also take the inverse values from those reported in the second column of Table 1 if, during the exchange between v and - v, the angle ω' is still imposed in the same reference frame R', without then offering this possibility to the angle ω of the reference frame R. A discussion on the symmetry or asymmetry of the two points of view on the two reference frames can be found in Guy [7].

4.3. Group Structure

We will discuss later on the question of the group structure of the complete transformation which integrates the γ factor and the offset in origins. The composition of several offsets is especially important when the different directions of relative movement are not parallel. If we stick to the γ factor that is a scalar, we do not have at its strict level any composition problem; we check that we have the group structure, i. e. that it is possible to find a v_3 such that $\gamma(v_3) = \gamma(v_1 \oplus v_2)$, where ⊕ is the law of relativistic addition of the speeds of modules v_i. One will check on various examples that $\gamma(v_3) = \gamma(v_1 \oplus v_2) = \gamma(v_1).\gamma(v_2)(1 + v_1v_2)$, in particular for parallel and perpendicular transport as defined above. When a velocity v_2 is expressed as dx'/dt' in the moving reference frame at velocity v_1 (where x' and t' are given by a usual Lorentz transformation) the expression of the velocity composition is independent of the g factor.

5. The Writing of the Lorentz Transformation in the Most General Case

We do not repeat here the demonstration of the complete derivation of the Lorentz transformation in the general case, nor do we examine its proper functioning in the context of our new proposal, postponing this necessary examination to a later work. For the moment, we are content to propose expressions that are consistent with the "geometric" choices made and with the whole process. In doing so, we follow a direction of reasoning that is the opposite of the usual direction where the factor γ appears as a consequence of the complete derivation. We have proposed a factor γ first. Equipped with this factor, expressing the distortions between durations (or lengths), the complete Lorentz transformations also involve origin offsets for the space and time coordinates in the relative movement of the reference frames at velocity v. This approach in the sense: γ factor → Lorentz transformation was described in [18]. It leads to a transformation involving on the one hand the space coordinates x, y and z, and on the other hand the coordinates of the photon useful for the definition of time, namely t_x, t_y and t_z. In the end, we have to manipulate a vector with 6 components (x, y, z, t_x, t_y, t_z). For space coordinates, transformations are written:

$$\vec{r'} = \gamma(\vec{r} - \vec{v}t) \qquad (7)$$

What is developing in:

$$x' = \gamma(x - v_x t) \qquad y' = \gamma(y - v_y t) \qquad z' = \gamma(z - v_z t) \qquad (8)$$

In (7), it is the module or scalar t that measures the displacement of R' relative to R in the vector vt, where v is the vector.

For time coordinates, it is necessary to take into account the different directions of t and t' (intervention of ωi and ω'i), but also the orientation of v with respect to t' and t' (intervention of ωi and ωi). We then have the following equations, where v is a scalar:

$$t'_x = \frac{\omega'_x}{\omega_x}\gamma\left(t_x - \frac{vx}{c^2}\frac{\omega_x}{\alpha_x}\right)$$

$$t'_y = \frac{\omega'_y}{\omega_y}\gamma\left(t_y - \frac{vy}{c^2}\frac{\omega_y}{\alpha_y}\right)$$

$$t'_z = \frac{\omega'_z}{\omega_z}\gamma\left(t_z - \frac{vz}{c^2}\frac{\omega_z}{\alpha_z}\right) \qquad (9)$$

For which we do not have a synthetic vector expression: we could write

$$\vec{t'} = \gamma\left(\mathrm{Proj}(t,t')\vec{t} - \frac{v}{c^2}\mathrm{Proj}(v,t')\vec{r}\right) \qquad (10)$$

subject to defining two projection operators of t on t' and v on t', Proj(t, t'), a function of ω_i and ω'_i, and Proj(v, t'), a function of ω'_i and α_i (as discussed in the next section).

In the simplified two-dimensional case of space (x, y), already discussed, the previous transformations are written by replacing ω_x, ω_y, by cosω and sinω, and α_x and α_y by cosα and sinα ; so for t'_x and t'_y as a function of t_x, x, and t_y and y, we have :

$$t'_x = \frac{\cos\omega'}{\cos\omega}\gamma(t_x - \frac{vx}{c^2}\frac{\cos\omega}{\cos\alpha})$$

$$t'_y = \frac{\sin\omega'}{\sin\omega}\gamma(t_y - \frac{vy}{c^2}\frac{\sin\omega}{\sin\alpha}) \quad (11)$$

In the case where α = ω and therefore ω = ω', we find the formulas already given in [6,7] which we noted:

$$\vec{t}' = \gamma(\vec{t} - \frac{v}{c^2}\vec{r}) \quad (12)$$

developed in :

$$t'_x = \gamma(t_x - \frac{v}{c^2}x) \quad t'_y = \gamma(t_y - \frac{v}{c^2}y) \quad t'_z = \gamma(t_z - \frac{v}{c^2}z) \quad (13)$$

With $v(v_x, v_y, v_z)$ of module v. The scalar time is built by $t^2 = t_x{}^2 + t_y{}^2 + t_z{}^2$ and can be enrolled in a four-component vector (x, y, z, t), from the six-component vector (x, y, z, t_x, t_y, t_z).

5.1. Commentary

In all the previous equations, the expression of the γ factor is given by the general formula (2). Established in our two-dimensional reasoning, this relation is valid for three dimensions as we have said. The angle δ is then defined in the plane (v, c') and its value as a function of the directions of the vectors in the three-dimensional space is given in Appendix 2 as recalled above.

The formulations given in [6,7] Guy deserve discussion. For the space coordinates, we wrote vector terms in vt, where the vector is t and the scalar v, unlike what we said just now in the relation [7]. This is written vt_x, vt_y, vt_z in the developed expressions. The latter are accurate because, in our past work, we have asked for parallelism between vectors v and c and c', or between v, t and t'; this leads to $v_x t = vt_x$, $vy_t = vt_y$, $vz_t = vt_z$. But the old relations are not valid in general: it is necessary to write $v_x t$ and not $v_t x$, because t is the parameter that counts (and not such or such t_x) to move along v, that we project in $v_x t$ etc. This explanation reconciles the two points of view (compare to other authors who have used a three-dimensional parameter to define time [19, 20].

6. Composition of Non-Collinear Lorentz Transformations

What can we say now about the composition of several reference frame displacements? It is now necessary to conceive this composition in a general way, within the framework of a time understood itself as a displacement, and by using its projections according to the various useful directions. When composing any velocity displacements, we will have to correlate oriented changes of standards, associated with changes in velocity directions. We can divide the problem into two sub-problems.

The first is set by the directions in which the clock photon is seen from the different reference frames; it is assumed that the choice of the reference frame where this direction is imposed has been made beforehand, the other directions being deduced from this, taking into account the relative speeds of the reference frames. This problem was addressed in the case of two reference frames, and projections useful for connecting standards were highlighted in the relation (10). The angles α, ω and ω'appearing in the developed relations (9) show the various projections to be made on the coordinate axes to refer to comparable terms, taking into account the different orientations of the vectors concerning times and displacements.

The second sub-problem concerns the actual composition of the movements of the reference frames at speeds v, v', v''' etc. The two problems are intertwined, but in order not to complicate things and limit ourselves to the composition of movements, we decide to neglect the first problem, or, more exactly, to assume that it does not arise; this is another way of saying that we have taken the different δ equal to zero, or that we have already made the projections announced in a relation such as (10). Otherwise, these projections will have to be added to those we will see later. Let's call (C) this choice (the δ = 0). If we limit ourselves to the composition of the scalars γ, we have seen above that the group structure was acquired (the difficulty arises for the offsets of the origins mixing "vector" spaces and times from different directions).

Let us therefore look at how we can compose two Lorentz transformations (as defined in the previous section) along any two directions v and v', knowing that the vectors t and t' of the first and second reference frame are parallel to v, and that the vector t'' of the third reference frame is parallel to v'', according to the choice (C). The following text, up to but excluding Section 7, is from Guy [6], with some modifications. The relations we have given can be written in a six-dimensional space for vectors

$m = (r, t) = (x, y, z, t_x, t_y, t_z)^T$ with $t = (t_x^{\,2} + t_y^{\,2} + t_y^{\,2} + t_z^{\,2})^2$

nd m' of analogous expression as a function of r' and t' (the exponent "T" means that a transposition is performed). Transformation is written:

$$m' = L(v).m$$

by continuing to use the notation L(v). In reality, we are dealing with a single space of dimension three counted twice, r and t (r' and t') both belonging to it. In the first reference frame, time is defined by the propagation of a signal along a certain direction t, which also makes it possible to define the ruler used to measure distances; t and v have the same direction (choice (C)). If the direction v is changed and displacements along a new direction v' are considered, length and time standards (marked by the movement of the mobile marking time) along this new direction must be considered; thus all useful vectors (having both temporal and spatial significance) must be projected onto the new direction before writing the Lorentz transformation linking the two reference systems. Let us designate by m_v (r_v, t_v) the vectors considered according to the direction v and by $m_{v'}$ ($r_{v'}$, $t_{v'}$) their projections according to v' (all the components of m are subjected to the projection operation). Let us call Proj(v,v') the matrix giving the projection of a vector considered along the direction v' on the direction v'. It is a diagonal matrix whose coefficients are ratios of the direction cosines of the directions v and v'. Without changing the notation, we consider that Proj(v,v') can be written in a two-times three-dimensional space, repeating the coefficients a second time have a 6-dimensional matrix. We check that these projection matrices commute with the matrices L. Thus, when passing from v to v' we write:

$$m_{v'} = Proj(v,v')m_v$$

The Lorenz transformation concerns two vectors m and m', where the components are measured using the same direction of the standards; for the first transformation we therefore have:

$$m'_v = L(v).m_v$$

To deal with a new Lorentz transformation according to v', we must first express the previous relation along this new direction:

$$m'_{v'} = Proj(v,v').L(v).m_v$$

before expressing the new transformation by:

$$m''_{v'} = L(v')m'_{v'}$$

By projecting everything onto the initial direction v by

$$m''_v = Proj(v',v).m''_{v'}$$

it finally comes

$$m''_v = L(v').\ L(v).m_v$$

where the property that the matrices L and Proj commute is used, as well as the identity Proj(v',v). Proj(v,v') = 1. We see in total that, by composition, we have:

$$L(v'') = L(v).L(v') \qquad (13)$$

in vector writing. So we can project in any direction we want. We can compose the vector relations giving r' and t' as a function of r and t then r'' and t'' as a function of r' and t'. Each time v and v' multiply a vector, the modules are written. The speed composition law is then:

$$v'' = v' \oplus v = \frac{v + v'}{1 + \frac{|v|.|v'|}{c^2}} \qquad (14)$$

where v and v' are vectors of modules |v| and |v'|. Each component behaves like a scalar and projects on the coordinate axes (taking into account the remarks given above that $t.v_x = t_x.v$ etc. - choice (C)). We obtain, for example for the component along the x-axis:

$$v''_x = \frac{v_x + v'_x}{1 + \frac{|v|.|v'|}{c^2}} \qquad (15)$$

and so on for the other components.

These new relations, thanks to a better symmetry between "vector" time and position, allow us to find an internal composition law and an abelian group structure. There is no need to add rotation after the fact. The proposed coordinate transformation and velocity composition formulas do not pose the same problems as non-collinear velocity compositions and Thomas rotation. We will verify that, in total, the proposed transformations and their compositions do not present any of the difficulties discussed in Guy [6].

7. What about Natural Phenomena? Conclusions

What is the relation between the previous formalism and physical reality? Every physical system carries its own clock, a way of talking about the tracking allowed by the movements of particles and photons that occur within it. The proper "rhythm" of this clock is accessed through the measurements made on the "at rest" system that the observer has in front of him. If the system is now moving relative to the observer, it will be understood after the previous calculations that the times associated with the starting system are modified, "dilated" or not. A wide variety of movements can be imagined, both for the interplay of phenomena within the system and for the relative movements of the system in relation to the observer. Undoubtedly, the complicated movements within the system of interest to us are difficult to reduce to a single parameter ω', which rather represents a kind of average of different movements (likewise does the standard Lorentz transformation reflect an average behavior?).

The quantitative values of temporal distortions, which can be calculated by a factor such as γ, are then varied: there is no generality and work is to be done in each case. We have given only one new way of posing problems from which to examine at new expense the interpretations already given in the literature (and sometimes subject to debate): from the examination of relativistic muons (whose lifetime is seen to be longer in their displacement than in their resting state) to that of clocks embedded in aircraft and for which the time distortion seems to vary according to the direction of rotation around the globe. We will also have to look at what is derived from the mathematical form of the γ factor on the structure of other physical relations (relating to energy for example): do we need a finer representation of the γ factor, function of what we know about the system? It is because we have spatialized time, that is, closely associated with a movement, that we have been able to make the proposals contained in this text. A number of points, detailed in the foregoing lines, deserve further elaboration. This article illustrates a new perspective on the speed of light, which plays a fundamental role in astronomy and astrophysics. What matters in physics, what we ultimately measure, are ratios such as v/c or c/v, not c alone. In another paper [21], we were motivated to assign it a different value on a cosmological scale and discuss a number of astrophysical issues.

Acknowledgements

We thank Guy Tsabary (Israel; co-author of the article by Tsabary and Censor quoted) for his encouragement to write this text. And many thanks to Bentlee H and the EAA team for their invitation and trust, and for their work preparing the text (an unpublished version appeared in 2019 on the Hal preprint repository).

References

1. Guy B. (2016) Linking quantum mechanics and general relativity? Reflections and proposals, <hal-01873420>.
2. Guy B. (2011) Time and space are of the same stuff, <hal-00651429>.
3. Guy B. (2017a) What can a better coupling between space and time concepts bring to thermodynamics ? <hal-01529570>.
4. Guy B. (2017b) General relativity and the epistemology of space and time: a relational approach, <hal-01529574>.
5. Guy B. (2019) *Space = Time. Dialogue on the system of the world*, Penta Editions, Paris.
6. Guy B. (2010) Les relations de Lorentz et le temps: proposition d'utilisation d'un paramètre tri-dimensionnel défini par un déplacement. La question du temps en physique.http://archive.org/details/LesRelationsDeLorentzEtLeTempspropositionDutilisationDun
7. Guy B. (2015) On the comparative age and aging of two twins (relativity theory), <hal-01196320>.
8. Lorentz H.A. (1904) Electromagnetic phenomena in a system moving with any velocity smaller than that of light, *Proceedings Acad. Sci. Amsterdam*, t. VI, 809-831.
9. Poincaré H. (1905) Sur la dynamique de l'électron, C. R. Acad. *Sc. Paris, 140*, 1504-1508.
10. Einstein A. (1905) On the electrodynamics of moving bodies, *Annalen der Physik, XVII*, 891-921.
11. Lévy-Leblond J.M. (1976) One more derivation of the Lorentz transformation, Am. *J. Phys. 44*, 3, 271-277.
12. Rougé A. (2002) *Introduction to relativity*, Les éditions de l'Ecole polytechnique, 180 p.
13. Guy B. (2013) On the "speed" of light and its measurement: disappearance of space and time standards; the standard of motion; <hal-00814874>; and communication to the 22nd General Congress of the French Physical Society, Marseille, July 2013 (P082).
14. Elbaz (1984) L'onde stationnaire et la transformation de Lorentz, C. R. Acad. Sc. *Paris, 298*, II, 13, 543-546.
15. Hoffman B. (1980) *Histoire d'une grande idée, la relativité*, Belin, Paris, 190 p.
16. C.W. Chou, D.B. Hume, T. Rosenband, D.J. Wineland (2010), Optical clocks and relativity, *Science*, 329, 1630-1633.
17. McCarthy (1999) The classical / newtonian derivation of Lorentzian equations for sound and other media processes, *J.*

New Energy, 3, 4, 54-61.
18. Guy B. (2004) *Lightning and thunder. Walks between space and time*. Editions EPU, Paris, 224 p.
19. Franco J.A. (2006) Vectorial Lorentz transformations, *Electronic Journal of Theoretical Physics, 9*, 35-64.
20. Tsabary & Censor (2005) An alternative mathematical model for special relativity, *Nuovo Cimento B*, 120, 2, 179-196.
21. Guy B. (2024) A diamond universe, *Int. J. Fund. Phys. Sci., 14*, 2, 22-40.

Appendix 1: Relation Between the Angles ω, ω' and α

The three angles ω, ω' and α, and the ratio β = v/c are related to each other. The trigonometric relations in any triangle are applied here to the POO' triangle (see Fig. 2). The angles and lengths of the sides are shown in Figure 3 and the relations are written:

$$\frac{ct'}{\sin(\omega - \alpha)} = \frac{vt}{\sin(\omega' - \omega)} = \frac{ct}{\sin(\pi + \alpha - \omega')}$$

We specify ω' and α, and we want to know ω. Equality is derived from the second and third terms:

$$\sin(\omega' - \omega) = v/c \sin(\pi+\alpha-\omega') = v/c \sin(\omega' - \alpha)$$

Hence

$$\omega' - \omega = \text{Arcsin}\{v/c \sin(\omega' - \alpha)\}$$

Hence

$$\omega = \omega' - \text{Arcsin}\{v/c \sin(\omega' - \alpha)\}$$

Which we can also note (taking into account the definitions of β and δ):

$$\omega = \omega' - \text{Arcsin}\{\beta \sin\delta\} \qquad \text{(A1)}$$

Appendix 2: Calculation of the Angle δ as a Function of the Direction Cosines of Vectors v and c' (in the Case of Three Spatial Dimensions)

Let α_x, α_y and α_z be the direction cosines of vector v, and ω'_x, ω'_y and ω'_z those of vector c'. The cosine of the sought angle δ is equal to the scalar product of the vectors v and c' divided by the product of their modules vc. We derive δ from it by the relation :

$$\delta = \text{Arc}\cos\sqrt{\alpha_x\omega'_x + \alpha_y\omega'_y + \alpha_z\omega'_z} \qquad \text{(A2)}$$

Citation: GUY, B. (2025). The Lorentz Transformation, Time and Space. Generalization of the γ Factor as a Function of the Direction of the Hidden Movement in the Clocks. *Explors Astron Astrophys, 1*(2), 1-13.