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Validation Of a Modified Normalized Kruskal-Wallis Fractal Dimension for Characterizing Pore Structure in Khuff Carbonate Reservoirs, Central Saudi Arabia

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Abstract

Carbonate samples were collected from surface exposures of the Khartam Reservoirs within the Permo–Triassic Khuff Formation of central Saudi Arabia. Porosity and permeability measurements were obtained for all samples, and pore-network heterogeneity was investigated using two independent fractal approaches derived from capillary-pressure data. The first method utilized the relationship between log-normalized pore radius, Log(R/Rmax), and Log10(water saturation), whereas the second employed Log(modified normalized Kruskal–Walli's parameter, MNKW) versus Log10(water saturation). In both methods, the fractal dimension was determined from the positive linear coefficient of the fitted quadratic expression according to Df = 2 + b. The calculated fractal dimensions ranged from approximately 2.36 to 2.86 and showed a strong positive association with permeability and pore-network connectivity. Porosity ranged from 2.269-11.253%, whereas permeability ranged from 0.264–3.445 mD. Representative samples with low, average, and high fractal dimensions yielded Df values of approximately 2.36, 2.72, and 2.86, respectively. Corresponding coefficients of determination increased from about 0.826 to 0.995 and 0.9997, indicating that fractal behavior becomes increasingly pronounced as pore-system complexity increases. Comparison of the two approaches demonstrated remarkable consistency. Regression analysis of 26 paired Df values produced a slope of 0.99531, an intercept of 0.00641, and R^2 = 0.9999986, confirming near-perfect agreement between methods. Bland–Altman analysis revealed a small mean bias (~0.0063 Df units) and narrow limits of agreement; however, a strong proportional bias was identified, indicating that differences between methods increase systematically with increasing Df. The results demonstrate that fractal dimension is an effective quantitative descriptor of pore-structure heterogeneity in tight carbonate reservoirs. Both approaches provide robust and highly consistent characterization of pore systems and may be extended to other porous materials, including nanomaterials, ceramics, catalysts, and engineered porous media.

Keywords: *Modified Normalized Kruskal-Wallis, Fractal Dimension, Khuff Formation, Carbonate Reservoir, Pore Structure, Permeability, Capillary Pressure, Water Saturation, Pore-Network Heterogeneity S*

Abbreviation Meaning

DF: Fractal Dimension
MNKW: Modified Normalized Kruskal–Wallis
R^2: Coefficient of Determination
MD: Millidarcy
SW: Water Saturation
R: Pore radius
RMAX: Maximum por radius
K: permeability (mD).
M: Slope of linear regression.
C: Intercept of Linear regression.
A: Quadratic coefficient.
B: Linear coefficient.

1. Introduction

Characterization of heterogeneous porous materials remains a major challenge in reservoir engineering and materials science. Fractal geometry provides a quantitative description of pore- network complexity and heterogeneity. Fractal dimensions derived from capillary-pressure data have been widely used to characterize porous media and predict reservoir quality. Fractal geometry has become an important tool for describing the complexity and heterogeneity of porous materials. The relationship between capillary pressure and wetting-phase saturation in fractal porous media has been extensively investigated and has provided a foundation for reservoir characterization [1]. Experimental verification of permeability estimation from capillary-pressure data demonstrated the applicability of fractal approaches in porous systems [2]. A theoretical relationship between capillary pressure and resistivity index based on fractal scaling theory was later proposed by [3]. The fractal dimension of pore-space geometry has been successfully applied to characterize pore-volume distribution and permeability in sandstone reservoirs [4]. Reported that heterogeneous pore-throat distributions are associated with higher fractal dimensions in sandstone reservoirs [5]. More recently, fractal approaches have been successfully applied to carbonate reservoirs, tight sandstones, and multiscale pore-network characterization, demonstrating strong correlations between fractal dimension, permeability, and pore connectivity [6-8]. Studies have independently demonstrated the applicability of electric potential energy, electric potential gradient, electric power and pressure head for estimating fractal dimensions in Shajara sandstone reservoirs [9-13]. However, these investigations were conducted separately despite being based on the same geological formation and petrophysical dataset. Recent advances in fractal characterization have expanded the understanding of pore-scale heterogeneity in sandstone reservoirs. Deep-learning-assisted fractal analysis has been successfully employed for quantitative characterization of tight sandstone pore systems, revealing strong relationships between pore morphology and fractal dimension [14]. Fractal dimensions have also been shown to be effective indicators of pore-throat heterogeneity, permeability, and reservoir quality in tight sandstone formations [15-17]. Studies integrating high-pressure mercury intrusion, NMR, micro-CT, and digital rock physics have demonstrated that multiscale fractal dimensions provide valuable information regarding pore connectivity and fluid transport behavior [18-22]. Furthermore, pore-structure evolution and fractal heterogeneity have been investigated under various geological and engineering conditions, including waterflooding, CO_2–water–rock Interactions, and freeze–thaw cycles, confirming the versatility of fractal methods in porous-material characterization [23-25]. These developments support the application of electrical-property-based fractal dimensions as Complementary descriptors of pore architecture and reservoir heterogeneity. The novelty of this study lies in introducing and validating a modified normalized Kruskal–Wallis (MNKW) fractal approach and demonstrating its equivalence to the conventional pore-radius scaling method for characterization of carbonate pore-network heterogeneity.

2. Method

Fractal dimensions provide a powerful quantitative framework for reservoir characterization. Two independent approaches were utilized to investigate the internal pore structure of Khartam reservoirs within the Permo-Triassic Khuff Formation, central Saudi Arabia. The first approach is based on a conventional reservoir parameter, namely normalized pore radius and water saturation. The second approach is based on a statistical parameter termed the Modified Normalized Kruskal–Wallis (MNKW) index. The logarithm of fractional water saturation, X = log10(Sw), was used as the independent variable. A second-order polynomial regression was fitted using the Excel LINEST function according to.

2.1. Conventional Log(R/Rmax) Fractal Approach

$$Log10\left(\frac{R}{Rmax}\right) = a[Log10(Sw)]^2 + bLog(Sw) + c \quad (1)$$

2.2. MNKW Fractal Approach

$$Log10(MNKW = a[Log10(Sw)]^2 + bLog10(Sw) + c \quad (2)$$

3. Results

Where a is the quadratic coefficient, b is the linear coefficient, and c is the intercept. The fractal dimension was subsequently calculated from the linear coefficient according to.

Df = 2 + b

The coefficients a, b, and c were obtained directly from polynomial regression using the LINEST function in Microsoft Excel. Fractal dimension is related to logarithm permeability as follow.

$$Df = m * Log(K) + c \quad (3)$$

Df= Fractal dimension.
K= permeability (mD).
M= Slope of linear regression.
C= Intercept of Linear regression.

Increasing pore-network heterogeneity is associated with increasingly robust fractal scaling behavior, and the modified normalized Kruskal–Walli's approach provides fractal-dimension estimates that are statistically equivalent to those obtained from the conventional Log(R/Rmax) method, although a small proportional bias exists across the measurement range. The bias can be explained by the fact that, MNKW parameter incorporates cumulative distribution characteristics of the capillary-pressure curve, whereas the Log(R/Rmax) approach is based directly on pore-radius scaling. This behavior indicates that the MNKW approach becomes progressively more sensitive to pore-system complexity at higher fractal dimensions, likely reflecting its dependence on cumulative saturation-distribution characteristics. Small differences in sensitivity to large pore throats may therefore become amplified at higher fractal dimensions. Increasing fractal dimension reflects greater pore-size variability, improved pore-throat connectivity, and increased geometric complexity of the pore network. Porosity ranged from 2.269-11.335%, whereas permeability ranged from 0.264–3.445 mD. Fractal dimensions varied from approximately 2.36 to 2.86 and increased systematically with permeability. The relationship between fractal dimensions derived from the conventional Log(R/Rmax) approach and those obtained from the modified normalized Kruskal–Wallis (MNKW) method is illustrated in Figure 1. Representative samples covering the full range of pore-network heterogeneity demonstrate that both methods yield highly comparable fractal-dimension estimates. The close correspondence between the two approaches provides preliminary evidence that the MNKW parameter effectively captures pore-structure complexity in a manner consistent with the established pore-radius scaling methodology.

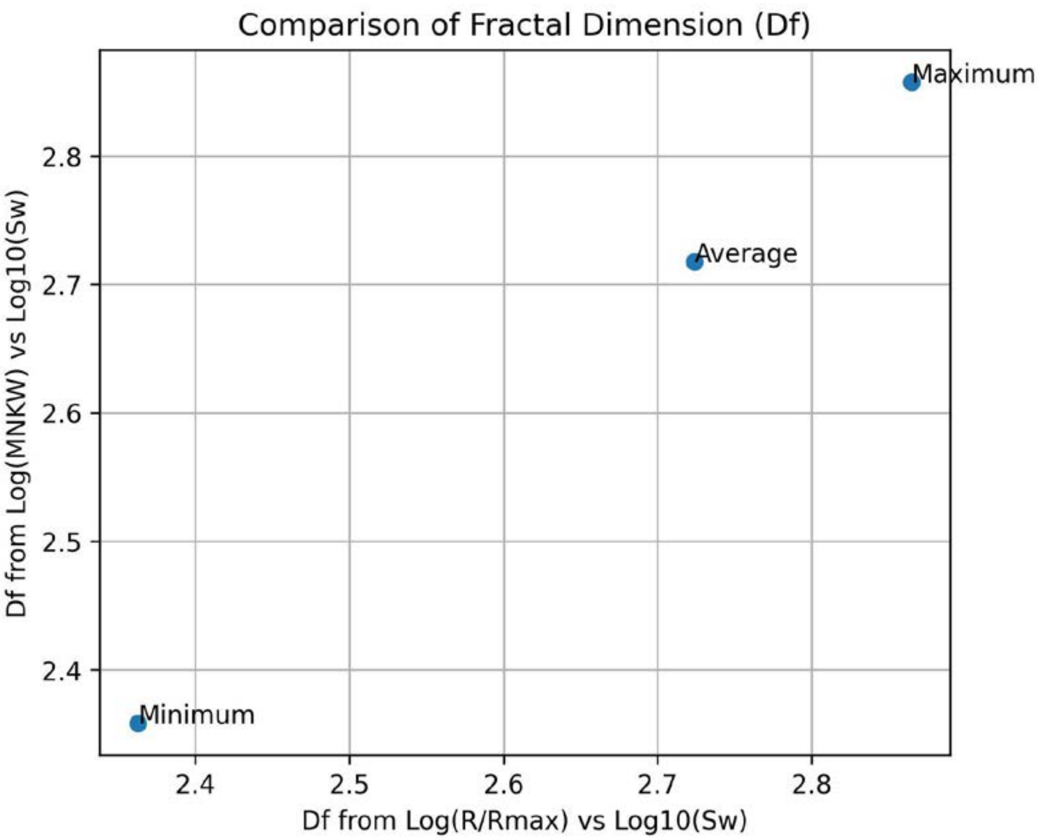

Figure 1: Comparison of Fractal Dimensions (Df) Derived from the Log(R/Rmax) and Mnkw Methods for Representative Samples

4. Discussion

The study demonstrates three important findings

Excellent agreement between methods

Regression of the 26 paired Df values yielded

Slope = 0.99531

Intercept = 0.00641

R = 0.9999993

R^2 = 0.9999986

A regression analysis of the 26 paired fractal-dimension values further confirms the consistency between the two approaches. As shown in Figure 2, the regression line closely follows the line of equality, yielding an almost unit slope and an exceptionally high coefficient of determination. These results indicate that the MNKW-based method reproduces the fractal dimensions obtained from the conventional Log(R/Rmax) method with near-perfect accuracy across the investigated range.

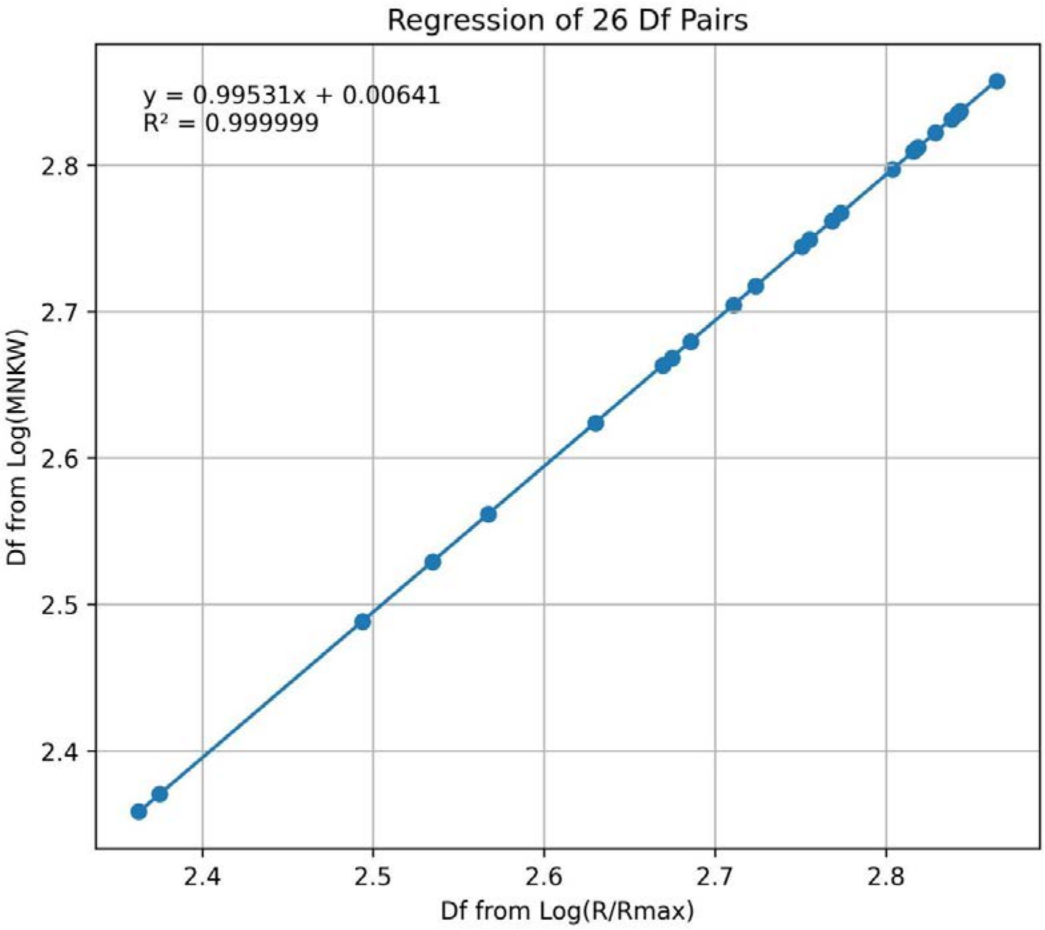

Figure 2: Regression of 26 Paired Fractal-Dimension Values Showing Near-Perfect Agreement Between Methods

These statistics indicate that the MNKW-based method reproduces the Log(R/Rmax)-based fractal dimensions with virtually perfect linear correspondence.

Increasing fractal dimension corresponds to stronger fractal behavior

Minimum-Df sample:

Df ≈ 2.36

$R^2 \approx 0.826$

Average-Df sample:

Df ≈ 2.72

$R^2 \approx 0.995$

Maximum-Df sample:

Df ≈ 2.86

$R^2 \approx 0.9997$

This trend suggests that highly heterogeneous pore systems exhibit more robust fractal scaling behavior than less heterogeneous systems.

Presence of proportional bias

Mean bias ≈ 0.0063 Df units.

Limits of agreement are narrow and all observations lie within them. However, the difference between methods increases systematically with increasing Df.

Difference-versus-mean regression:

Slope = 0.00470

R = 0.9696

$R^2 = 0.9401$

To further evaluate the agreement between the two methods, a Bland–Altman analysis was performed. Figure 3 shows the distribution of differences between the MNKW-derived and Log(R/Rmax)-derived fractal dimensions as a function of their mean value. Although the mean bias is small and all observations fall within the limits of agreement, a statistically significant proportional bias is evident. The magnitude of the difference between methods increases systematically with increasing fractal dimension, suggesting a slightly greater sensitivity of the MNKW approach in highly heterogeneous pore systems.

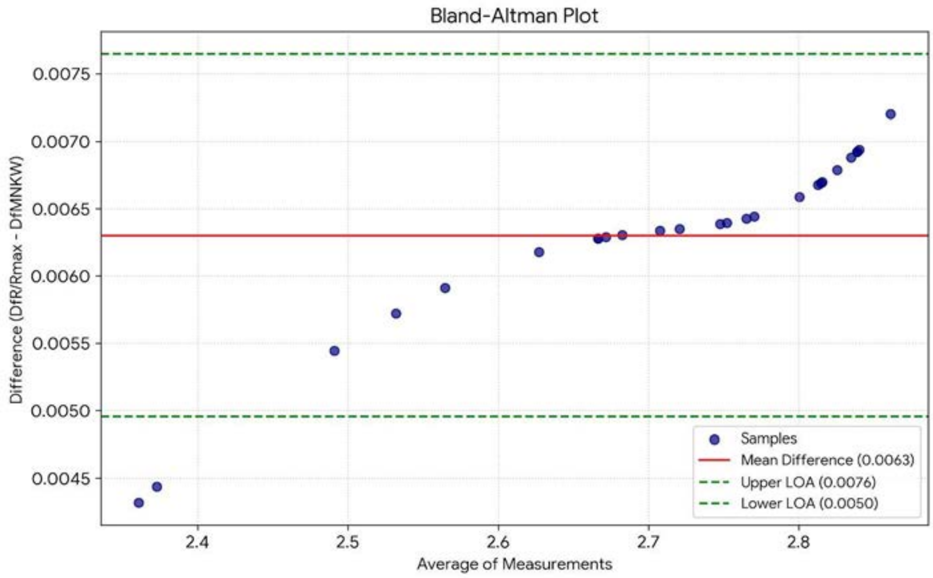

Figure 3: Bland–Altman Plot Showing Bias and Limits of Agreement Between the Two Methods. Therefore, the Two Methods Show Excellent Agreement but are Not Strictly Interchangeable Across the Entire Df Range Without Accounting for Proportional Bias

5. Conclusions

The modified normalized Kruskal–Wallis (MNKW) fractal approach provides fractal-dimension estimates that are in near-perfect agreement with those obtained from the conventional normalized pore-radius method. Fractal dimensions ranged from approximately 2.36 to 2.86 and increased systematically with permeability, indicating that greater pore-network heterogeneity is associated with enhanced pore-throat connectivity and fluid-flow capacity. Samples exhibiting higher fractal dimensions also displayed stronger fractal scaling behavior, reflected by increasing coefficients of determination. Although Bland–Altman analysis revealed a small proportional bias between the two approaches, the overall agreement was excellent, confirming the robustness of the MNKW method for pore-structure characterization. The results demonstrate that fractal dimension is an effective quantitative descriptor of pore-system architecture in tight Khuff carbonates and may serve as a valuable reservoir-quality indicator where conventional petrophysical parameters alone do not adequately capture pore-network complexity. Beyond carbonate reservoirs, the proposed methodology has potential applications in the characterization of a wide range of porous materials, including sandstones, ceramics, catalysts, nanomaterials, and engineered porous media. The observed increase in permeability with increasing fractal dimension suggests that pore-network connectivity exerts a stronger control on fluid-flow capacity than porosity alone in the investigated Khuff carbonate samples.

Author Contributions

Khalid Elyas Mohamed Elameen Alkhidir: Conceptualization, Methodology, Formal analysis, Investigation, Validation, Visualization, Data curation, Writing – original draft, Writing – review & editing, Resources, Project administration.

Conflict of Interest

The author declares no conflicts of interest.

References

1. Toledo, P. G., Novy, R. A., Davis, H. T., & Scriven, L. E. (1994). Capillary pressure, water relative permeability, electrical conductivity and capillary dispersion coefficient of fractal porous media at low wetting phase saturations. *SPE advanced technology Series, 2*(01), 136-141.
2. Li, K., & Horne, R. N. (2002, May). Experimental verification of methods to calculate relative permeability using capillary pressure data. In *SPE Western Regional Meeting* (pp. SPE-76757). SPE.
3. Li, K., & Williams, W. (2007). Determination of capillary pressure function from resistivity data. *Transport in porous media, 67*(1), 1-15.
4. Zhang, Z., & Weller, A. (2014). Fractal dimension of pore-space geometry of an Eocene sandstone formation. *Geophysics, 79*(6), D377-D387.
5. Wang, Z., Pan, M., Shi, Y., Liu, L., Xiong, F., & Qin, Z. (2018). Fractal analysis of Donghetang sandstones using NMR measurements. *Energy & fuels, 32*(3), 2973-2982.
6. Cheng, Y., Luo, X., Zhuo, Q., Gong, Y., & Liang, L. (2024). Description of pore structure of carbonate reservoirs based on fractal dimension. *Processes, 12*(4), 825.
7. Wang, W., Lin, C., & Zhang, X. (2024). Fractal dimension analysis of pore throat structure in tight sandstone reservoirs of Huagang Formation: Jiaxing area of East China Sea Basin. *Fractal and Fractional, 8*(7), 374.
8. Zhang, H., Guo, L., Wu, Z., & Ma, J. (2024). Pore-throat structure, fractal characteristics and permeability prediction of tight sandstone: The Yanchang Formation, Southeast Ordos Basin. *Scientific Reports, 14*(1), 27913.
9. Alkhidir, K. E. M. E. (2018a). Arithmetic relaxation time of induced polarization fractal dimension for

characterizing Shajara reservoirs of the Shajara Formation. Nanoscience and Nanotechnology, 2(1), 1–8. https://doi.org/10.5281/zenodo.20932853

10. Alkhidir, K. E. M. E. (2018b). Geometric relaxation time of induced polarization fractal dimension for characterizing Shajara reservoirs of the Shajara Formation of the Permo-Carboniferous Unayzah Group, Saudi Arabia. SciFed Journal of Petroleum, 2(1), 1–6. https://doi.org/10.5281/zenodo.20362693
11. Alkhidir, K. E. M. E. (2018 c). Resistivity fractal dimension for characterizing Shajara reservoirs of the permocarboniferous Shajara formation Saudi Arabia. *Int J Petrochem Sci Eng, 3*(3), 109-112.
12. Alkhidir, K. E. M. E. (2018d). Seismo-electric field fractal dimension for characterizing Shajara reservoirs of the Permo-Carboniferous Shajara Formation, Saudi Arabia. Petroleum & Petrochemical Engineering Journal, 2(2), 1–8. https://doi.org/10.23880/ppej-16000156
13. Al-Khidir, K. E. M. E. (2017). Pressure Head Fractal Dimension for Characterizing Shajara Reservoirs of the Shajara Forma-tion of the Permo-Carboniferous Unayzagh Group, Saudi Arabia. *Arch Pet Environ Biotechnol: APEB-113. DOI, 10,* 2574-7614.
14. Song, X., Feng, C., Li, T., Zhang, Q., Zhou, J., & Sun, M. (2025). Pore Structure Quantitative Characterization of Tight Sandstones Based on Deep Learning and Fractal Analysis. *Fractal and Fractional, 9*(6), 372.
15. Zhang, L., Liu, J., He, X., Feng, F., Li, W., Wang, M., ... & Zhu, Y. (2025). Fractal Characteristics and Influencing Factors of Pore Structure in Tight Sandstone: A Case Study from Chang 6 Member of the Southwestern Yishan Slope. *Processes, 13*(4), 988.
16. Jiang, Z., Liu, F., Cai, G., He, H., Shi, W., & He, Y. (2025). Fractal characterization of pore structure evolution in pre-fractured sandstone under cyclic freeze-thaw. *Geoenergy Science and Engineering, 247,* 213649.
17. Li, P. (2025). Fractal Analysis of Pore–Throat Structures in Triassic Yanchang Formation Tight Sandstones, Ordos Basin, China: Implications for Reservoir Permeability and Fluid Mobility. *Fractal and Fractional, 9*(7), 415.
18. Feng, L., Wang, R., Liu, N., & Gao, Z. (2025). Fractal characteristics of full-scale pores and throats in tight sandstone and their application in the analysis of the main controlling factors of a reservoir. *Frontiers in Earth Science, 13,* 1701181.
19. Xie, W., Yin, Q., Zeng, J., Wang, G., Feng, C., & Zhang, P. (2023). Fractal-based approaches to pore structure investigation and water saturation prediction from NMR measurements: A case study of the gas-bearing tight sandstone reservoir in Nanpu Sag. *Fractal and Fractional, 7*(3), 273.
20. Lin, W., Wu, Z., Li, X., Yang, Z., Hu, M., Han, D., ... & Zhang, J. (2022). Digital characterization and fractal quantification of the pore structures of tight sandstone at multiple scales. *Journal of Petroleum Exploration and Production Technology, 12*(9), 2565-2575.
21. Mao, W., Yao, Y., Qin, Z., Liu, Y., Han, J., & Liu, Z. (2024). Pore structure characterization of sandstone under different water invasion cycles using micro-CT. *Geomechanics and Geophysics for Geo-Energy and Geo-Resources, 10*(1), 53.
22. Cheng, Y., Luo, X., Hou, B., Zhang, S., Tan, C., & Xiao, H. (2023). Pore structure and permeability characterization of tight sandstone reservoirs: From a multiscale perspective. *Energy & Fuels, 37*(13), 9185-9196.
23. Wu, J., Gong, W., Wang, Q., He, Y., Guo, J., Bao, Y., ... & Li, R. (2025). Dynamic Characterization of Microscopic Pore Structure in Medium–High Permeability Sandstones During Waterflooding. *Nanomaterials, 15*(10), 747.
24. Yang, C., & Yang, E. (2025). Mineral composition and pore structure on spontaneous imbibition in tight sandstone reservoirs. *Scientific Reports, 15*(1), 7504.
25. Zhu, K., Li, K., Ji, Y., Li, X., Liu, X., Liu, K., & Chen, X. (2024). 3D characterization of pore structure and pore scale seepage simulation of sandstone based on computational tomography. *Water, 16*(7), 1022.